Co-funded by the European Union

MATH 3D GEO VR

Workshop Materials

for teachers and students

Mathematical models for teaching three-dimensional geometry using virtual reality

ENGLISH VERSION

Workshop materials for teachers and students "Mathematical models for teaching three-dimensional geometry using virtual reality"

Created by the Math3DgeoVR consortium.

Co-funded by the European Union

Co-funded by the European Union (Math3DgeoVR, project no. 2021-1-PL01-KA220-HED-000030365). Views and opinions expressed are however those of the author(s) only and do not necessarily reflect those of the European Union or Fundacja Rozwoju Systemu Edukacji. Neither the European Union nor the granting authority can be held responsible for them.

Lodz University of Technology
universidade de aveiro theoria poiesis praxis
UNIVERSITY OF ŽILINA
UNIVERSITY OF TARTU
UNIVERSITY OF SILESIA IN KATOWICE

Oculus Quest 2 1

Tutorial: Navigating the modules 7

Modules in the VR Application 10

Module 1: Trajectory 23

Module 2: Angles in a cuboid 30

Module 3: Angles in a pyramid 39

Module 4: Non-Euclidean geometry 46

Module 5: Maxima and minima of functions 50

Module 6: Systems of linear equations 56

Module 7: Prisms - sections of prisms, grids with prisms 62

Module 8: Pyramids – sections of pyramids, grids with pyramids 68

Module 9: Planetary system 75

Module 10: Exploring the Solar System 79

Module 11: Geometrical interpretation of partial derivatives 81

Module 12: Spherical coordinates 86

Module 13: Vectors, operations on vectors, scalar 92

Oculus Quest 2

Introduction to Oculus Quest 2

The Oculus Quest 2, now known as the **Meta Quest 2**, is a standalone virtual reality goggle that offers a number of advanced features. Here is a detailed description of their features and instructions on how to use them.

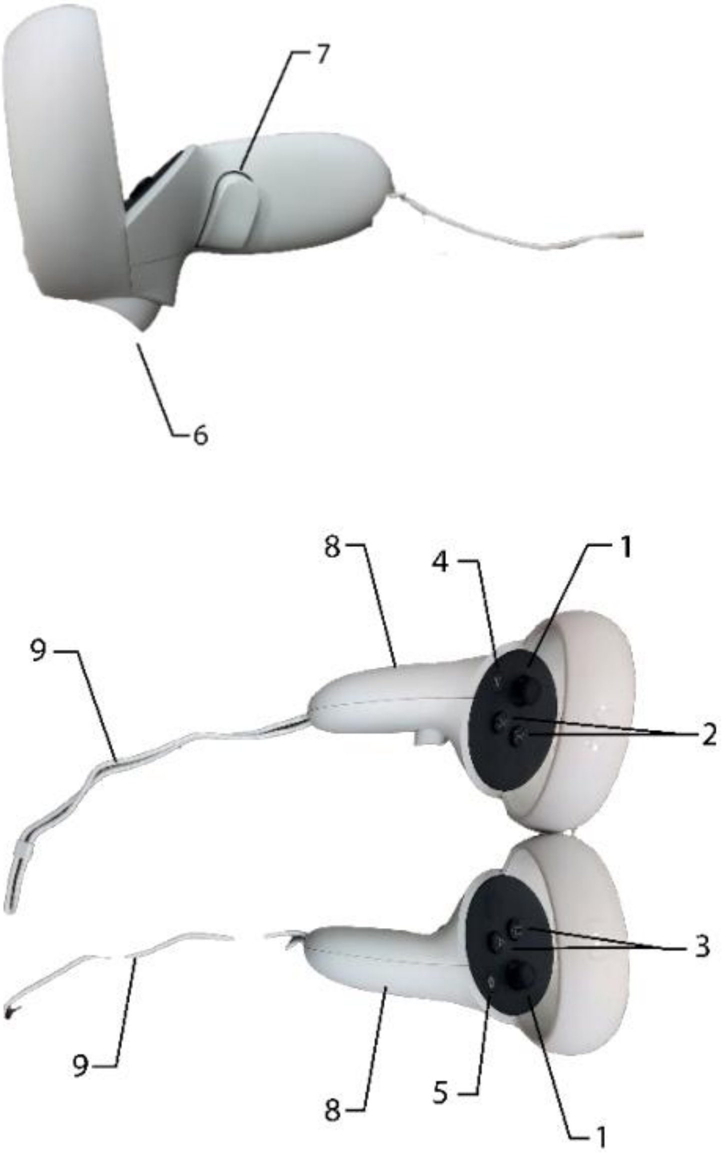
The Oculus Quest 2 consists of two main items:

1. Head-mounted display (HMD, Figure 1)
2. Touch Controllers (Figure 2)

1. Charging Port
2. Power Button
3. Volume Control
4. Audio Port
5. Lens Adjustment
6. Adjustment Straps
7. Adjustment Straps
8. Eyeglass Spacer (optional)

Figure 1. Head-mounted display

(photo adopted from Meta Quest 2 New User Guide, University of South Carolina. (n.d.). *[Image depicting the Meta Quest 2 headset]*. In *Meta Quest 2 New User Guide*. Retrieved from https://sc.edu/about/offices_and_divisions/cte/teaching_resources/docs/quest2_user_guide.pdf).



1. Joysticks
2. X/Y Buttons (Left Controller)
3. A/B Buttons (Right Controller)
4. Menu Button (Left Controller)
5. Oculus Button (Right Controller)
6. Trigger Buttons
7. Grip Buttons
8. Battery Compartments
9. Wrist Straps

Figure 2. Controllers

(photo adopted from Meta Quest 2 New User Guide, University of South Carolina. (n.d.). *[Image depicting the Meta Quest 2 headset]*. In *Meta Quest 2 New User Guide*. Retrieved from https://sc.edu/about/offices_and_divisions/cte/teaching_resources/docs/quest2_user_guide.pdf).

Basic features

Displays: The goggles are equipped with two displays with a resolution of 2064 x 2208 pixels per eye, providing a clear and detailed image.

Processor: The Quest 2 runs on a Qualcomm Snapdragon XR2 processor, which enables smooth operation of VR games and applications.

Motion Tracking: The goggles offer motion tracking in 3D space thanks to four cameras placed outdoors, allowing you to interact with your surroundings.

Controllers: Touch controllers are included for precise control in the virtual world.

Community Functions: Users can participate in multiplayer games and live events to enrich the VR experience.

PC Compatibility: The goggles can be connected to computers, allowing users to use more demanding VR applications..

How to start with the Oculus Quest 2

How to turn on the goggles

To turn on the Oculus Quest 2, you need to:

1. Press the power button on top of the goggles.
2. Wait for the system to boot up and display the Oculus logo.

Registering an account

In order to use Oculus Quest 2, it is necessary to have a Meta (formerly Facebook) account. The registration process includes:

1. Downloading the Oculus app: You can download the app from the App Store or Google Play store.
2. Logging in: The user must log in to their Meta account.
3. Configure profile: The user sets preferences, adds payment information and creates a PIN for the Oculus store.

Resetting the goggles

If there are problems with the operation of the goggles, you can reset them.

1. Reset to factory settings:
    - Turn off the goggles.
    - Press and hold the power button and the volume down button simultaneously for about 10 seconds.
    - When the Oculus logo appears, release the buttons.

- Use the volume buttons to navigate and select “Factory Reset”.

2. Application Reset: You can also reset the Oculus app on your mobile device, which can help solve connection problems.

The Oculus Quest 2 is an advanced device that offers a wealth of virtual reality options for gamers and experience seekers alike.

Oculus Quest 2 technical requirements

To take full advantage of Oculus Quest 2, your computer must meet certain technical requirements. Here are the most important of them:

- Processor: Intel Core i5-4590 or AMD Ryzen 5 1500X, or better
- RAM: Minimum 8 GB
- Operating system: Windows 10
- Graphics card: NVIDIA GeForce GTX 1060 or better, AMD Radeon RX 480 or better
- Ports: USB port available

Note that these requirements apply to using Oculus Link, which allows you to connect the goggles to a PC and play VR games from the Rift or Steam library. If you want to use the Questa 2 as a standalone device, without connecting it to a PC, the requirements are slightly less. It is also worth noting the length of the Oculus Link cable. It is recommended to use the original cable or good replacements to ensure optimal length and freedom of movement during gameplay.

Setup problems

Oculus Quest 2, despite its advanced features, can encounter various problems during use. Here are the most common ones and how to solve them.

During the initial setup, users may encounter several difficulties.

Stopping on updates

Goggles may fail to go through the update process. If this happens, try restarting the device or resetting it to factory settings if the problem persists [1].

Pairing code

You may be prompted to enter a pairing code. Open the Oculus app on your mobile device and follow the instructions to resume setup [1].

Software issues

Users may experience software issues such as:

Application crashes

Applications may sometimes hang or become unresponsive. If this happens, restarting the goggles or forcing a software update in settings [1,4] may help.

Black screen

Users may see a black screen after removing the goggles. In this case, simply restart the device to restore the normal view [4].

Performance problems

During heavy use, performance problems may occur.

Overheating

The goggles can get hot, especially when playing for a long time. If this happens, it's a good idea to take a break so the device can cool down [2].

Image quality problems

Users may notice that the image quality is not satisfactory. This may be due to inadequate settings or the need to update the software [2].

Health problems

Using VR goggles can lead to some health problems.

VR Disease

Some users may experience symptoms similar to motion sickness, such as dizziness or nausea. To minimize these symptoms, it is recommended to take breaks and avoid prolonged use [6].

Account problems

In case of problems with your Meta (Facebook) account:

Login Problems

Users may have difficulty logging into their account. It is worth checking that the login information is correct and that the Oculus app is updated [1].

In conclusion, the Oculus Quest 2 is an advanced device that can encounter various problems during use. Many of them can be solved by updating the software, restarting the device or resetting it to factory settings. In case of health problems, taking breaks from use is recommended.

References

[1] https://vrpolska.eu/poradnik-nowego-posiadacza-questa/
[2] https://mobiletrends.pl/sprawdzamy-gogle-oculus-quest-2-od-facebooka-czy-wprowadza-wirtulana-rzeczywistosc-pod-strzechy/
[3] https://www.youtube.com/watch?v=1uSoGOqmVbE
[4] https://business.oculus.com/support/444171669614375/?locale=pl_PL
[5] https://securecdn.oculus.com/sr/oculusquest-warning-polish
[6] https://motionsystems.pl/vr-sickness/
[7] https://www.meta.com/pl-pl/help/quest/articles/getting-started/getting-started-with-quest-2/what-is-meta-quest-2/
[8] https://www.meta.com/pl-pl/help/quest/articles/headsets-and-accessories/using-your-headset/

Tutorial: Navigating the modules

Controllers and Hands

You can do most of the interactions in this application using both the controllers and your own hands (if hand tracking is enabled in the VR goggles settings), to activate hand tracking put the controllers down so that they don't move and then place your hands within the VR goggles viewing range, to activate controller tracking simply take the controllers in your hands.

Hand tracking - to use hand tracking, it must first be enabled at the system level system menu -> quick settings -> settings -> device -> hands and controllers -> hand tracking.

Walking

By using the joystick on the left controller you can move forward backwards and sideways the front is always in the direction you are looking by swinging the joystick of the right controller to the left or right you can turn in increments of 45 degrees.

Teleport

By leaning the joystick of the right controller forward you activate the teleport indicator you point it anywhere and then take your finger off the joystick you will be moved to the indicated place. The arrows at the end of the pointer indicate your direction of view after teleportation you can adjust it by tilting the joystick knob sideways.

You can call the teleport indicator with your right hand by positioning , palms parallel to the ground straight index finger pointing forward straight thumb pointing to the left side approval of the teleport occurs when you point the straight thumb forward.

Grasping objects

Dip the tip of the controller into a graspable object and use the grasp button on the handle of the controller to grab this object you will hold it until you let go of the button you can also grab such an object with your hand and tighten all your fingers on it or grab it only with your index finger and thumb this action can be tested by grabbing the tablet levitating now on the left side of the door.

Interface controls

As you probably already know you can interact with interface elements by dipping your finger or the tip of the controller a tiny white ball, this also applies to various types of slider buttons.

Pop-up menu

To bring up the pop-up menu press the flat button on the left controller or Make a pinch gesture (with your index finger and thumb of your left hand) while keeping your hand raised and facing the VR goggles. In the same way, you can close this menu. The pop-up menu allows you to exit the module to the main menu, i.e. the current room, at any time, or to exit the application also allows you to change the volume of the language to switch between sitting and standing mode, and to show and hide the screen in the prompt.

Working with the Math3DgeoVR modules

Modules in the application are available modules corresponding to different mathematical issues for each of them there is an introductory part a test part and also examples of practical application. The main navigation panel is presented in Figure 3.

Start the module after completing the tutorial from the menu on this screen Select modules (Figure 4) and then press the button corresponding to the module.

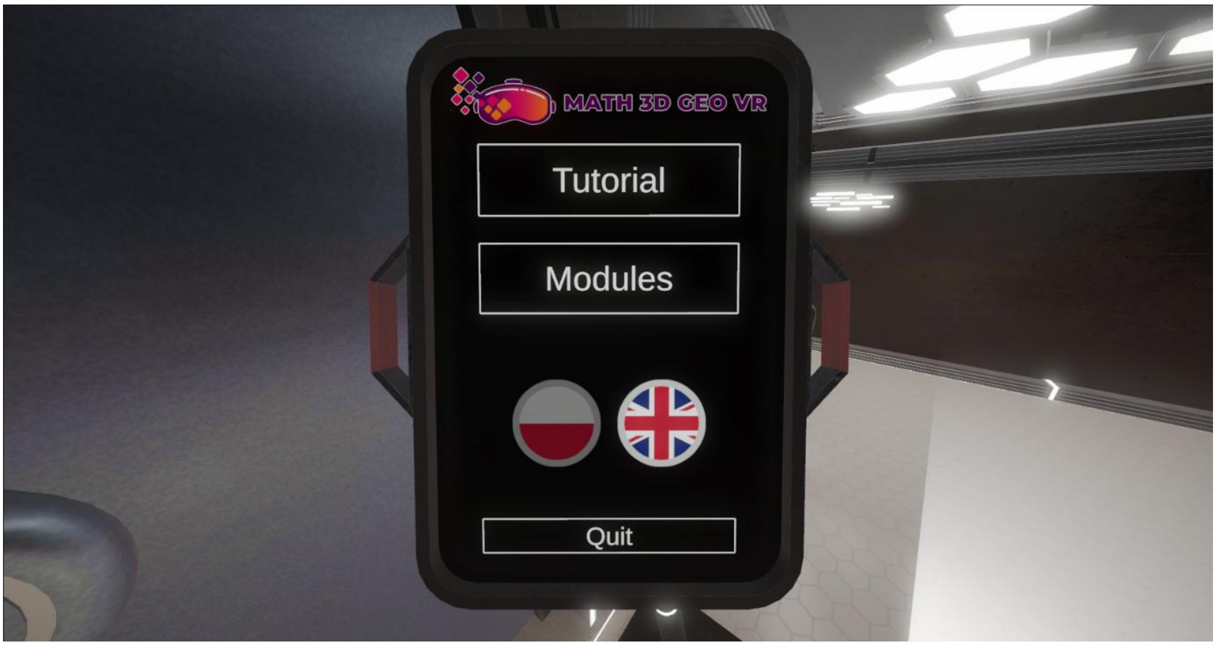


Figure 3. Main navigation panel in the Math3DgeoVR application.

Figure 4. Selection of VR modules in the Math3DgeoVR application.

Exit the modules

In most cases, you can exit the module to the main menu by pressing the button on the exit door at any time. You can also exit a module by pressing the bottom button on the pop-up menu.

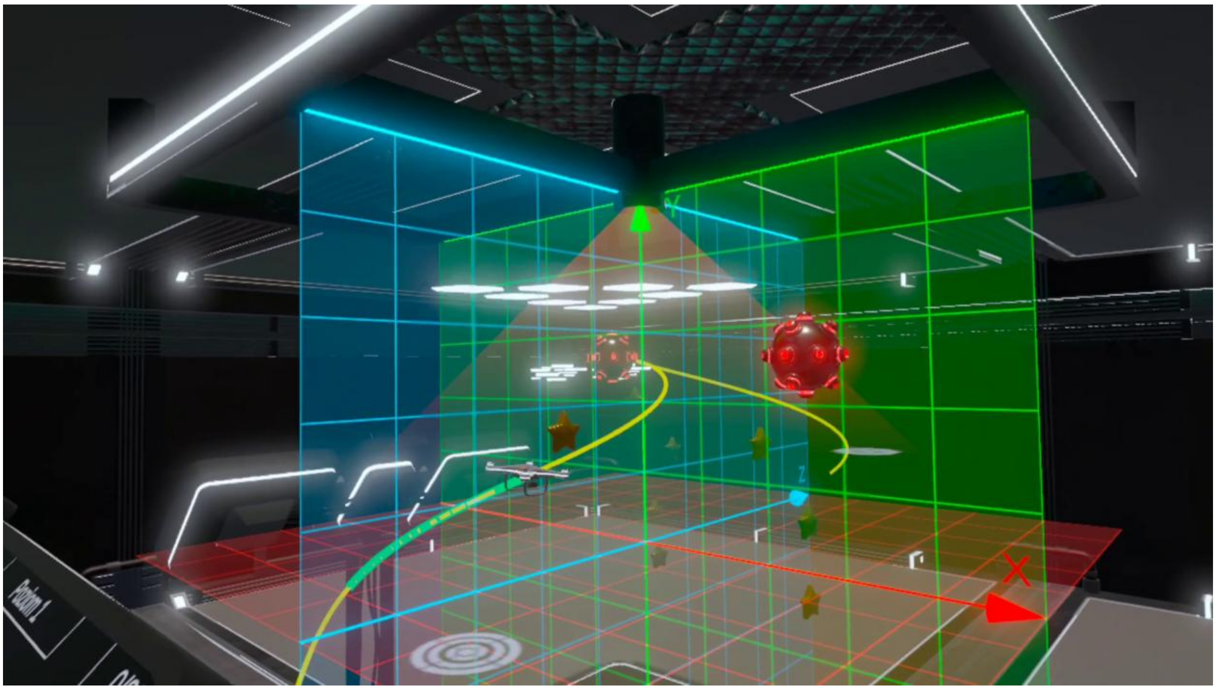
Tips-modules may contain additional screens with tips on, among other things, their specific controls to show or hide these screens press the B button on the right controller you can also change their visibility with the button in the pop-up menu.

Modules in the VR Application

Module 1: Trajectory

In this module, students will explore the relationship between mathematical functions and their graphical representations, focusing on spatial curves. The objective is to understand how a function of one variable can describe a three-dimensional curve, such as the trajectory of a moving object, like a drone. Students will design the flight path of a drone using two functions—one representing horizontal motion and the other representing vertical motion. The challenge is to navigate through specific points while avoiding obstacles. By manipulating the functions, students can visualize the drone's path in both 3D space and its projection on the XY plane.

The figure shows a hologram with a drone's flight trajectory.

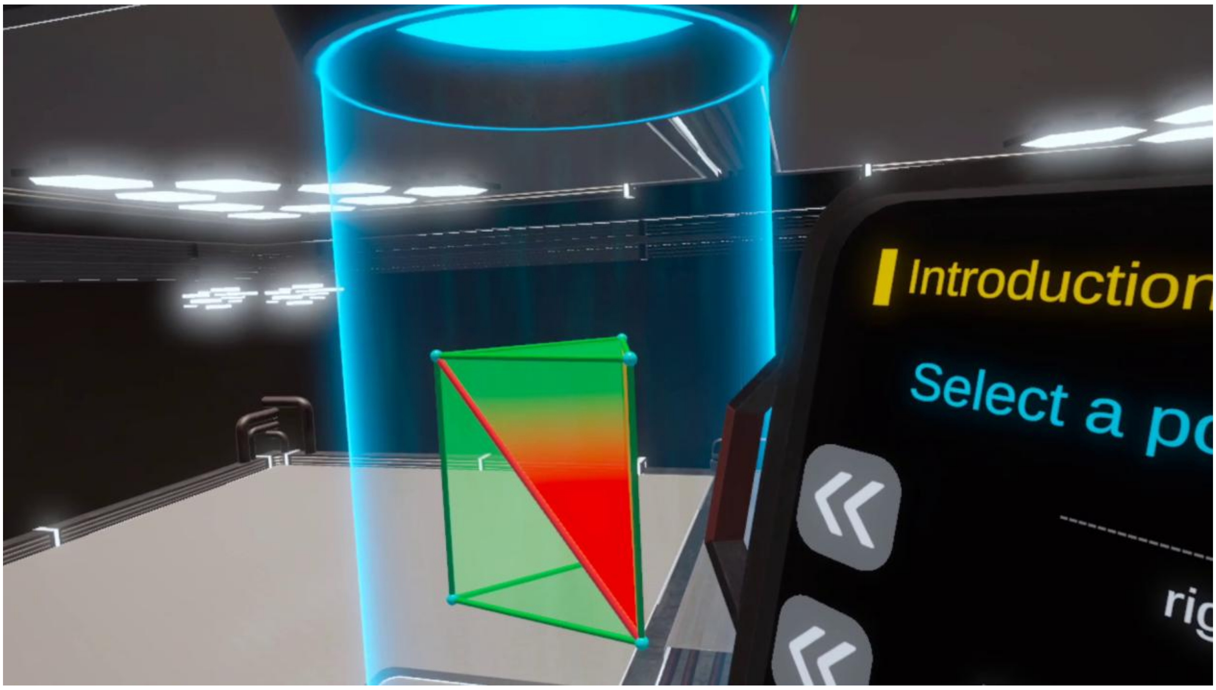


- **Lesson scenario 1:** Graphs of trigonometric functions of one variable
- **Lesson scenario 2:** A vector-valued function

Module 2: Angles in a prism

The topic "Angles in a prism" involves the analysis of angles formed by the diagonals and edges of a prism. A prism, being a three-dimensional geometric, is one of the fundamental objects studied in spatial geometry. Understanding the angles that form between various elements of a prism is crucial for a deeper understanding of solid geometry and its applications to real-world problems. In this module, you can get acquainted with solids and. A solid with an example of a given angle will appear on hologram you can take it out and look at it up close.

The figure shows a hologram with a triangular prism.

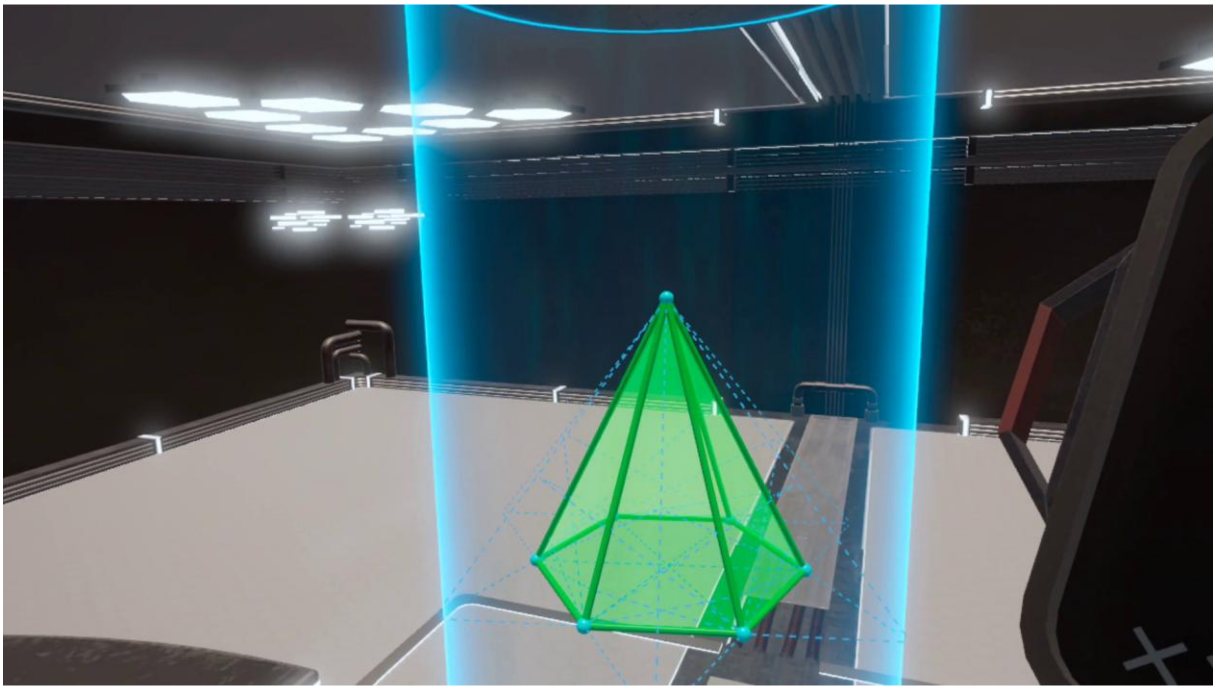


- **Lesson scenario 1:** Angles in a cuboid
- **Lesson scenario 2:** Calculation of edge length, surface area and volume in a cuboid

Module 3: Angles in a pyramid

In this module, students will learn to identify, calculate, and understand angles in pyramids by applying geometric principles. The setting is similar to the previous module on spatial curves, but now the focus shifts to analyzing and manipulating pyramidal shapes. Students will work with various pyramids, exploring different tasks using interactive functions such as learning mode, practice mode, and examples mode. Through this module, students will deepen their understanding of spatial geometry and develop the ability to calculate angles between faces, edges, and vertices of pyramidal solids.

The figure shows a hologram with a hexagonal pyramid.

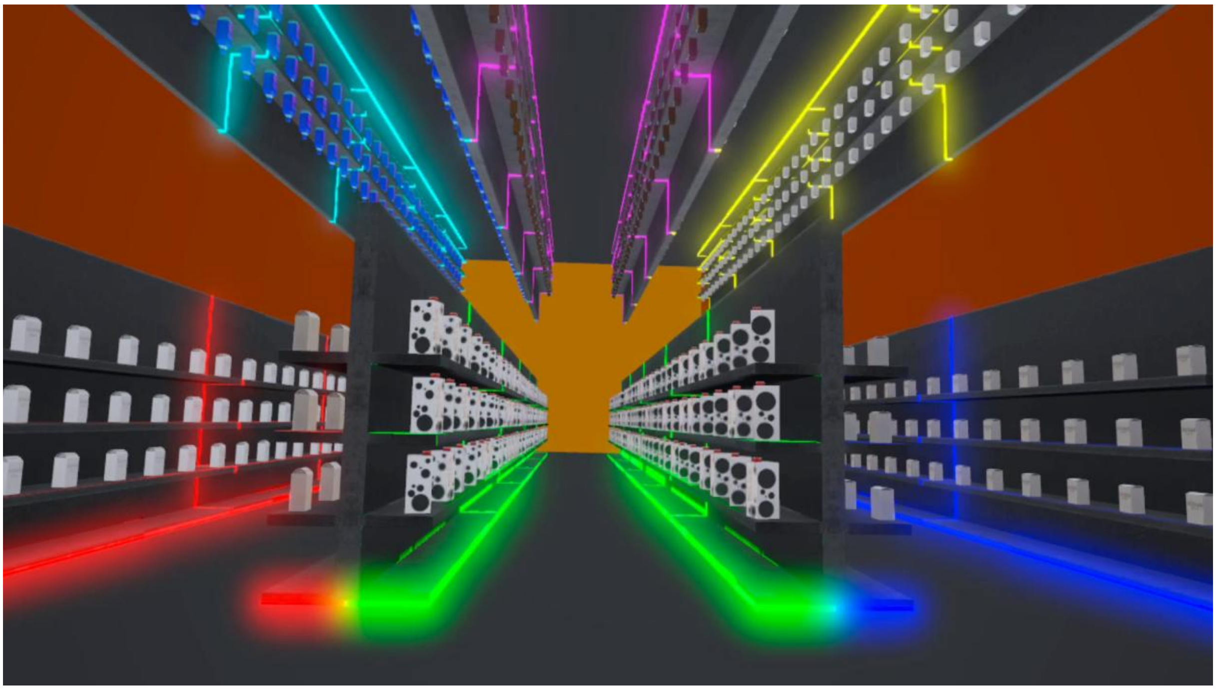


- **Lesson scenario 1:** Pyramids and total surface area
- **Lesson scenario 2:** Angles in a pyramid

Module 4: Non-Euclidean geometry

In this module, students will explore elliptic geometry, a branch of non-Euclidean geometry that rejects Euclid's fifth postulate, the parallel postulate. In elliptic geometry, any two lines intersect at some point, meaning the concept of parallel lines does not exist. This has profound implications for understanding shapes and distances in curved spaces, such as the surface of the Earth. The VR-based module allows students to experience elliptic geometry in practice by navigating through a building where the paths resemble ellipses. This hands-on approach helps students visualize and understand the properties and principles of non-Euclidean geometry in an immersive environment.

The figure shows a modeled non-Euclidean space – a “non-Euclidean grocery store”.

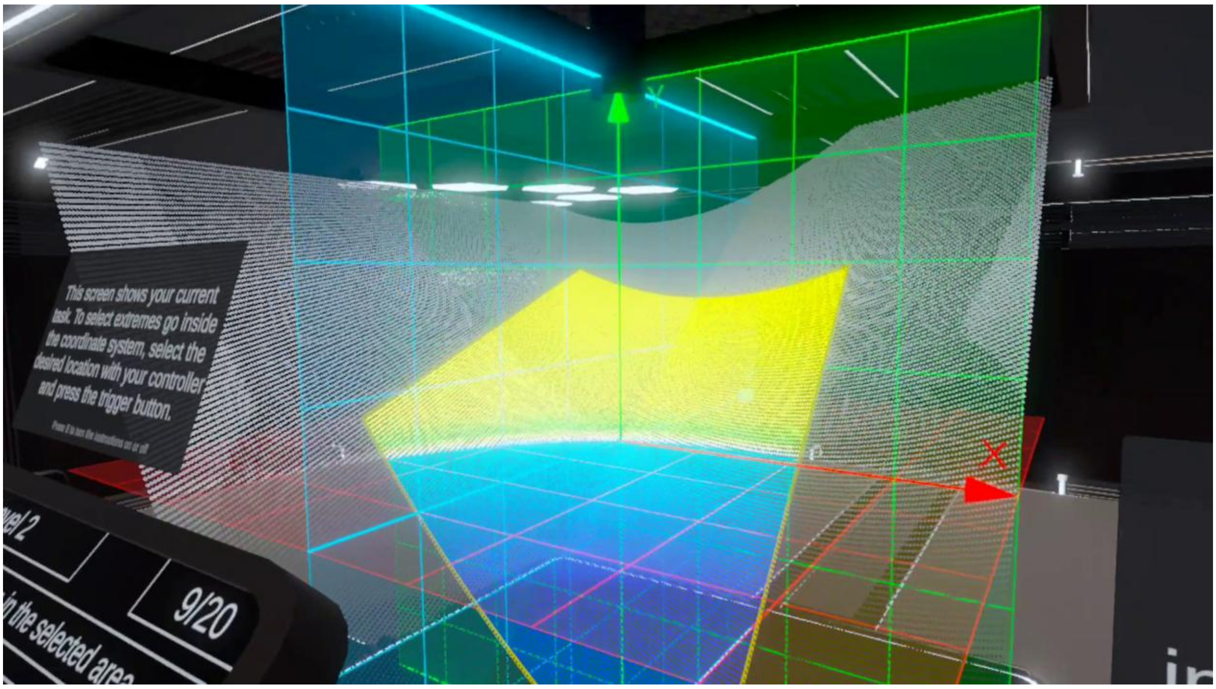


- **Lesson scenario 1:** Euclidean geometry
- **Lesson scenario 2:** Basics of non-Euclidean geometry

Module 5: Maxima and minima of functions

In this module, students will learn to find global extrema (both maximum and minimum values) of functions of two or three variables. The task is presented in an interactive way, where a system of three equations for the x, y, and z planes is displayed on a central screen. Students must identify the global extrema by placing markers (represented as spheres) on a 3D visualization of the surface generated by the equations. The module helps students understand how to interpret the geometry of functions and identify critical points where the function reaches its highest or lowest values globally, rather than just locally.

The figure shows a hologram with a function graph.

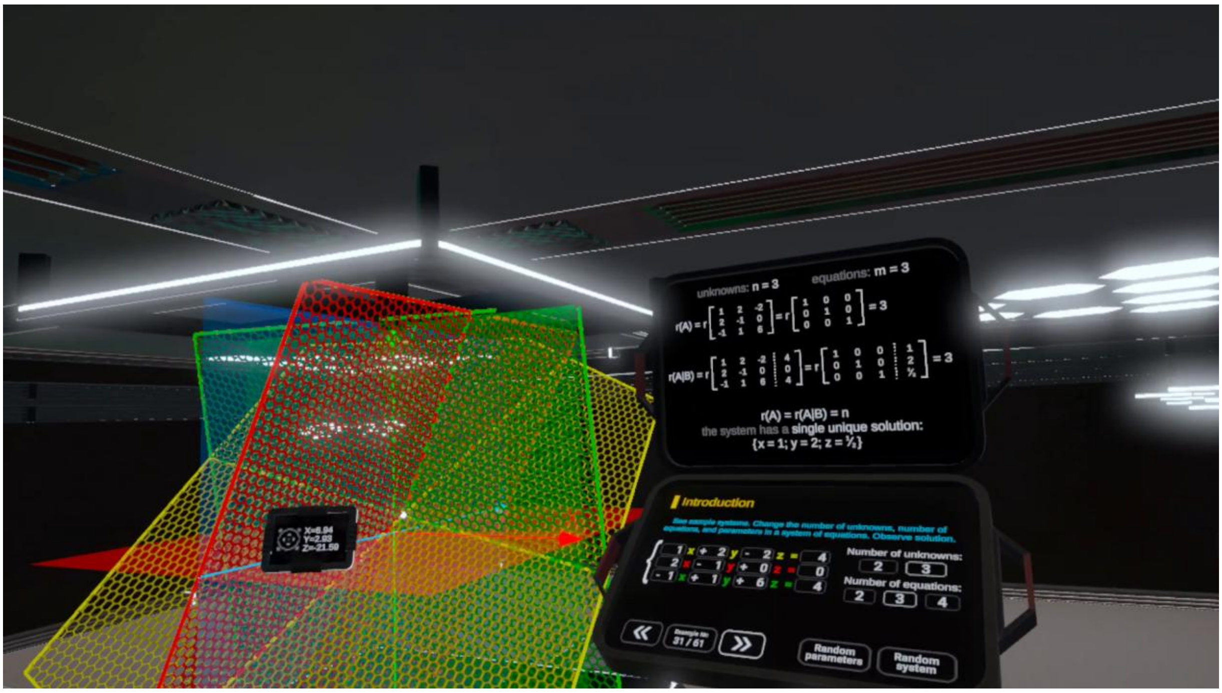


- **Lesson scenario 1:** Local minimum and maximum: definition, geometric interpretation
- **Lesson scenario 2:** Necessary and sufficient conditions for an extremum of a function of two variables

Module 6: Systems of linear equations

In this module, students will explore systems of linear equations through interactive visualizations. The main screen displays equations that students input using a tablet interface. From this tablet, students can choose from over 60 pre-made examples or modify parameters such as variables, equations, and coefficients. Additionally, they have the option to randomize the entire system or specific parameters like values for x, y, and z. Students can also adjust the number of unknowns or equations, providing a flexible environment for both beginner and advanced problem-solving. A secondary tablet displays matrices, determinants, and the solutions to these systems, offering students the opportunity to explore how linear algebra concepts apply to solving systems of equations.

The figure shows a hologram with a system of equations.

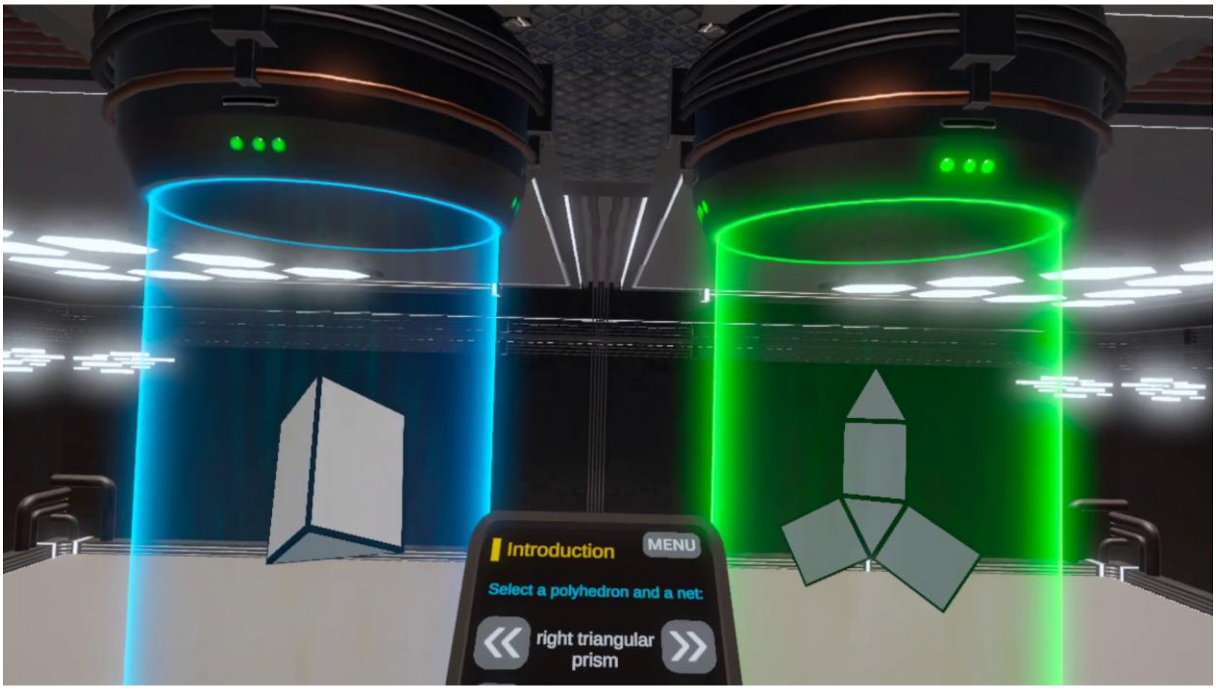


- **Lesson scenario 1:** Geometric interpretation of linear equations in spaces
- **Lesson scenario 2:** Solving linear equations

Module 7: Prisms

This module focuses on the geometry of prisms with a particular emphasis on understanding their spatial arrangement within grids. Students will with tasks involving grids of prisms and pyramids, visualizing how these solids interact in a structured arrangement.

The figure shows holograms with a prism and its grid.



- **Lesson scenario 1:** Grids of prisms
- **Lesson scenario 2:** Calculating the total surface area and volume of prisms

Module 8: Pyramids

This module focuses on the geometry of pyramids, with a particular emphasis on understanding their spatial arrangement within grids. Students will also engage with tasks involving grids of pyramids, visualizing how these solids interact in a structured arrangement.

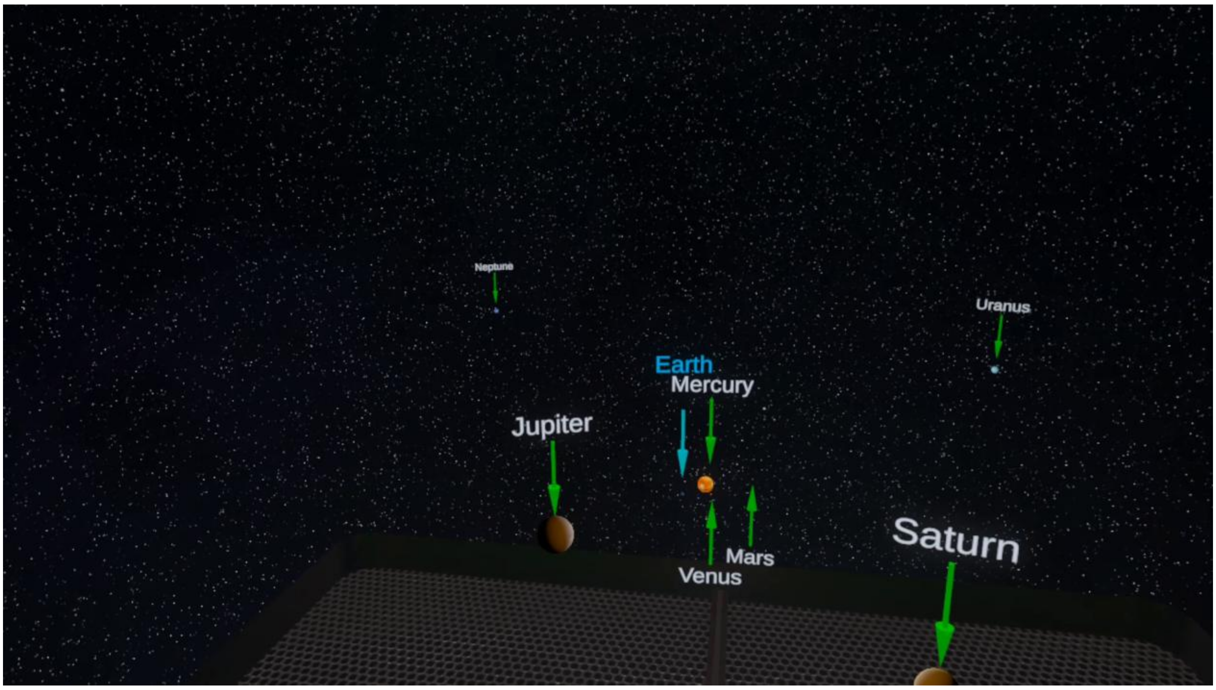
The figure shows holograms with a pyramid and its grid.

- **Lesson scenario 1:** Grids of pyramids
- **Lesson scenario 2:** Volume of a pyramid

Module 9: Planetary system

This module introduces students to the mechanics and geometry of planetary systems. Students will explore how planets orbit a central star, focusing on the interplay of forces, trajectories, and shapes of orbits. Using interactive tools, they will visualize the orbits of planets in 3D space and adjust parameters such as orbital radius, eccentricity, and velocity. The module emphasizes understanding the basic laws of planetary motion, such as those described by Kepler, without delving into overly complex mathematics. Students will see how orbits can be elliptical or circular and how gravity governs these motions.

The figure shows a visualization of the planets in the Solar System.



- **Lesson scenario 1:** Distances in the Solar System
- **Lesson scenario 2:** Quantities comparisons in the Solar System

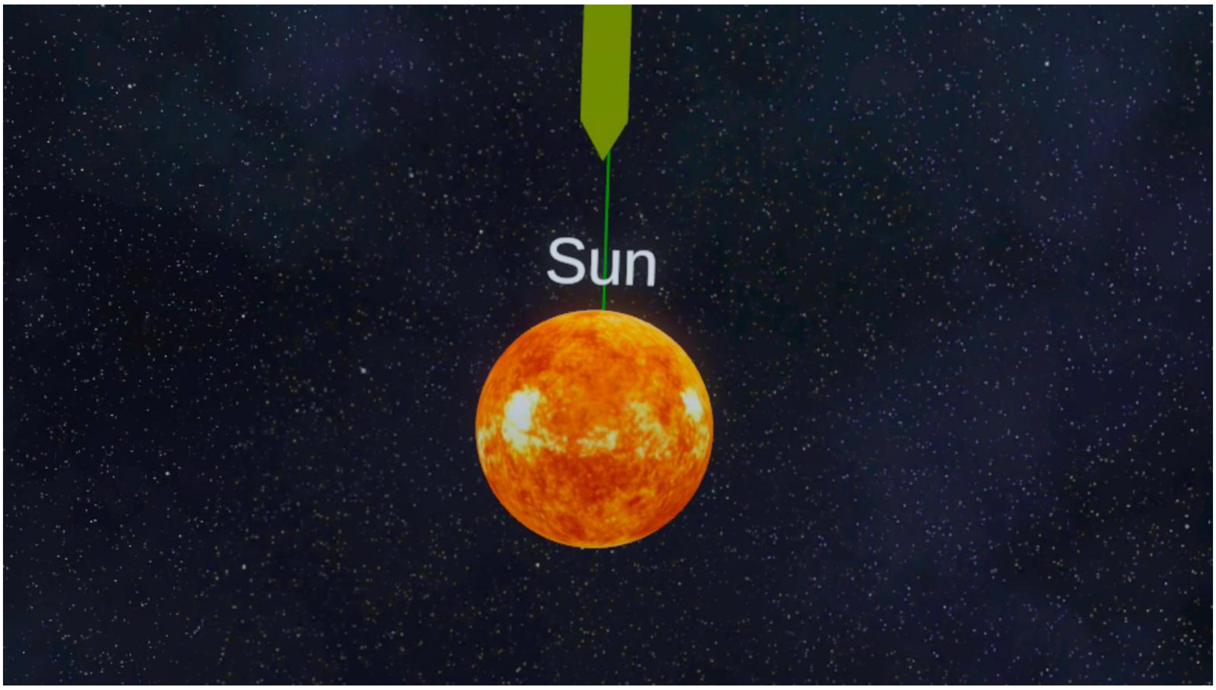
Module 10: Exploring the Solar System

This module introduces students to the topic of distance in space travel. Students will explore the Solar System by moving between planets using speeds known to mankind:

- the second cosmic velocity (or escape velocity),
- the highest speed during the Apollo 11 mission,
- the speed of the Parker Solar Probe,
- 1/100 of the speed of light,
- the speed of light.

Students will learn how long it will take to travel between planets and how it is affected by gravity. The journey from the Sun to the Earth at the speed of light takes over 8 minutes, and when we finally see our planet, it disappears in a moment. This shows how small the Earth is compared to the distance traveled.

The figure shows the Sun in the Solar System.

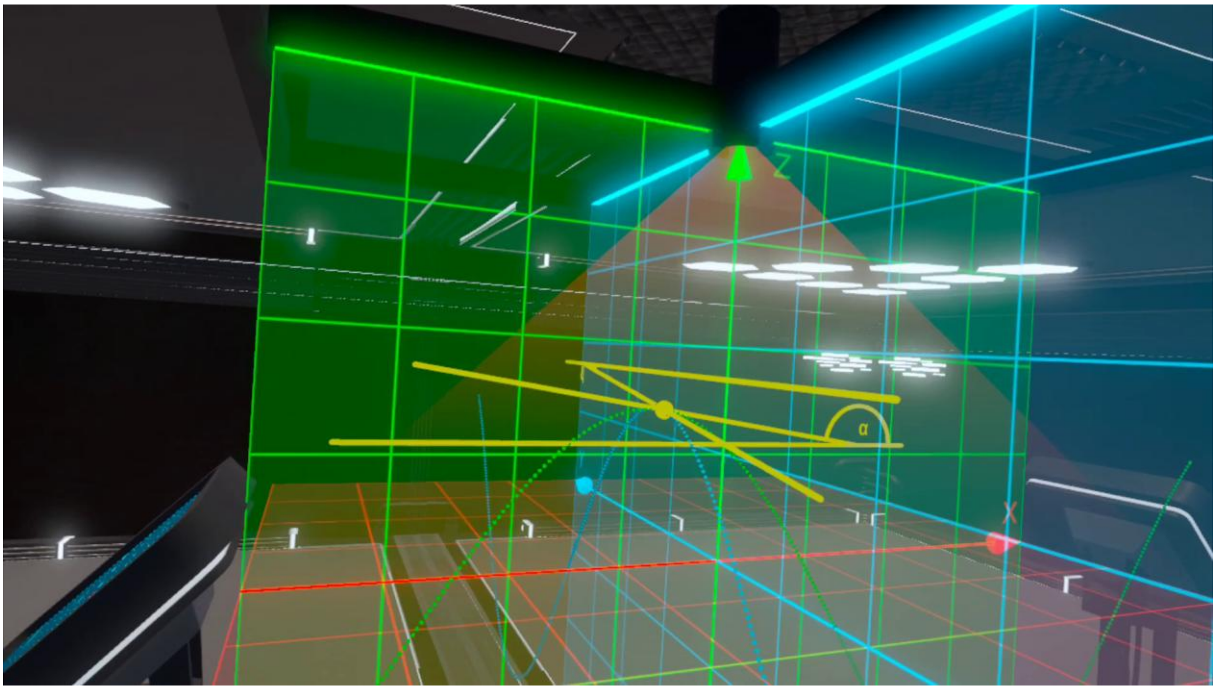


- **Lesson scenario 1:** Space exploration – basic concepts
- **Lesson scenario 2:** Conquest of space

Module 11: Geometrical interpretation of partial derivatives

In this module, students explore the geometrical meaning of partial derivatives in multivariable calculus. Directional derivatives represent the rate of change of a function in a specified direction, while partial derivatives measure changes along a single axis. Through interactive 3D visualizations, students will observe how the slope of a function varies depending on direction and position. The module allows students to manipulate surfaces and vectors to understand how these derivatives are computed and applied. This hands-on approach bridges the gap between abstract mathematical formulas and their real-world interpretations.

The figure shows a hologram with a graph of a function and a visualization of the partial derivative.

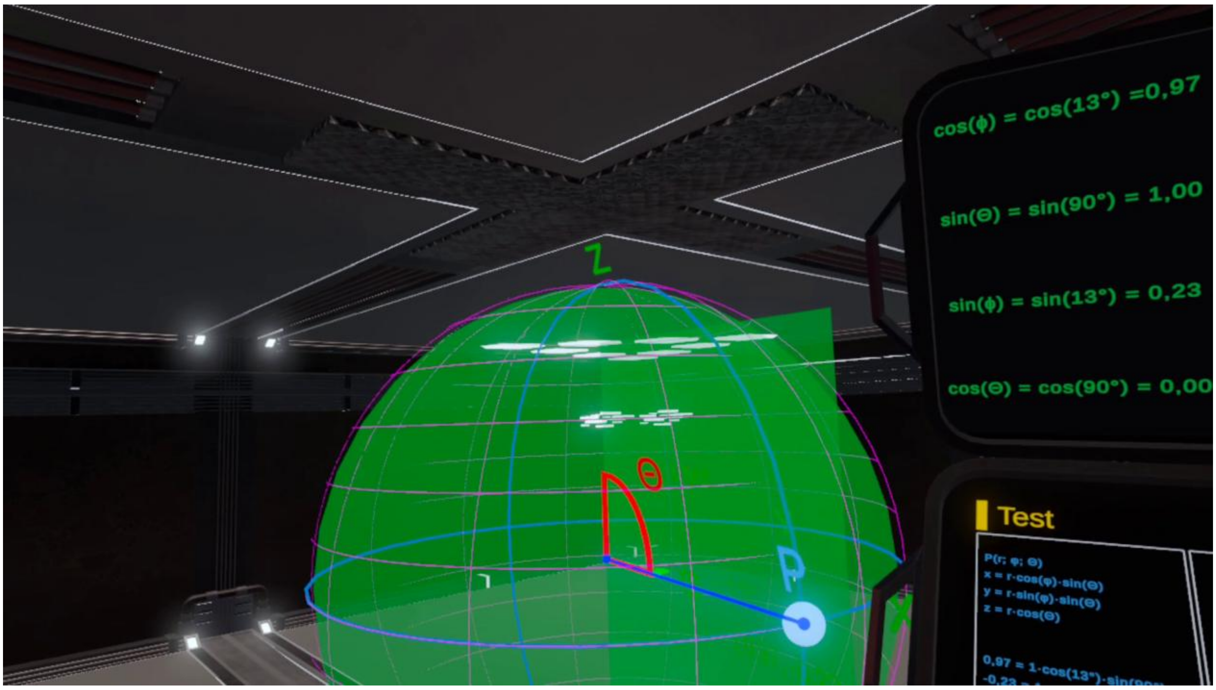


- **Lesson scenario 1:** Geometric interpretation of partial derivatives
- **Lesson scenario 2:** Calculating partial derivatives

Module 12: Spherical coordinates

In this module, students will explore the concept of spherical coordinates, a system used to describe points in three-dimensional space. Unlike Cartesian coordinates, spherical coordinates specify a point’s position using three values: the radial distance (r), the polar angle (θ), and the azimuthal angle (ϕ). This coordinate system is particularly useful for problems involving symmetry around a central point, such as in physics or engineering. The module includes interactive visualizations where students can manipulate these parameters to see how a point’s position changes in 3D space. Additionally, they will practice converting between Cartesian and spherical coordinates and solving problems that involve integrating functions over spherical regions.

The figure shows a hologram with visualization of spherical coordinates.

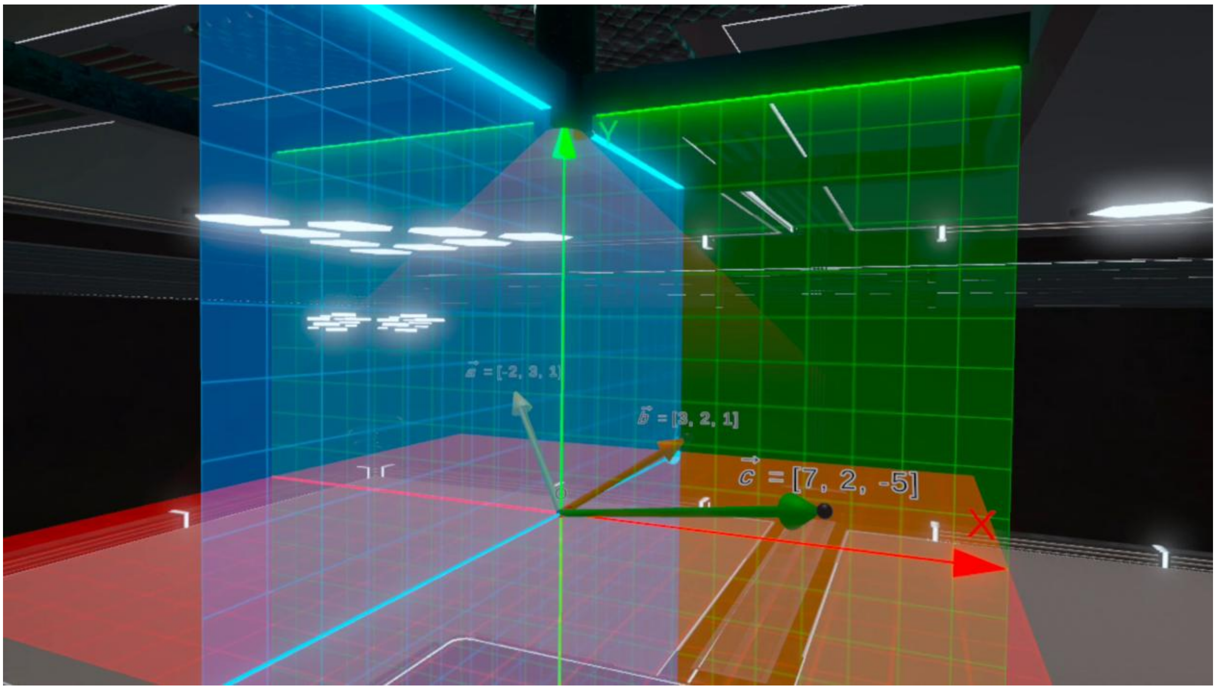


- **Lesson scenario 1:** Polar coordinates
- **Lesson scenario 2:** Spherical coordinates

Module 13: Vectors, operations on vectors

This module introduces students to vectors and fundamental operations performed on them. Vectors are mathematical objects with both magnitude and direction, making them essential tools for describing physical quantities and spatial relationships. Students will explore basic vector operations, such as addition, subtraction, scalar multiplication, and normalization, and learn how to compute the magnitude of a vector. The module provides interactive visualizations where students can manipulate vectors in 2D and 3D spaces, observe the effects of operations, and understand their geometric interpretations.

The figure shows a hologram with vectors in 3D space.



- **Lesson scenario 1:** Geometric interpretation of vectors in three-dimensional space, operations on vectors
- **Lesson scenario 2:** Scalar product, vector product in three-dimensional space

Module 1: Trajectory

Lesson scenarios with VR applications

Lesson scenario 1

Lesson title

Graphs of trigonometric functions of one variable

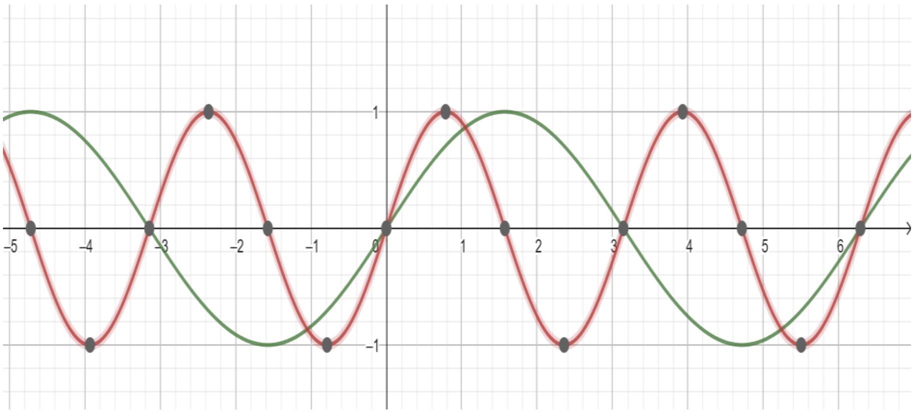
Learning outcomes

- Draws graph of function $f(x) = a\sin(x - b) + c$.
- Interprets the parameters a, b, c for the function $f(x) = a\sin(x - b) + c$.

The course of the lesson

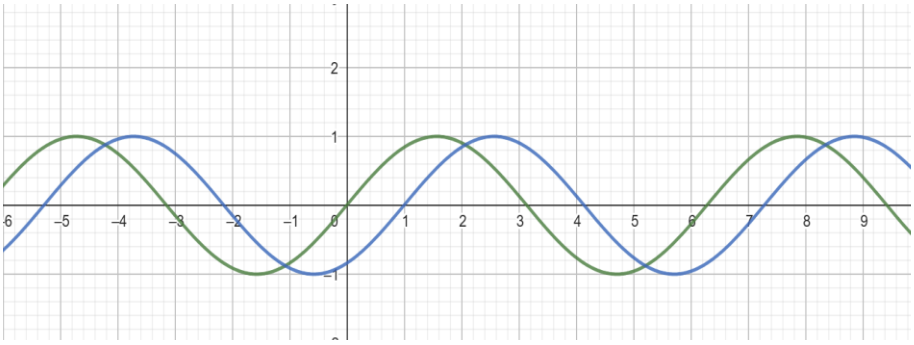
Step 1.

Draw the graphs of the functions $\sin(x)$ and $\sin(2x)$. Discuss how the coefficient a affects the graph of the function $f(x) = \sin(ax)$.



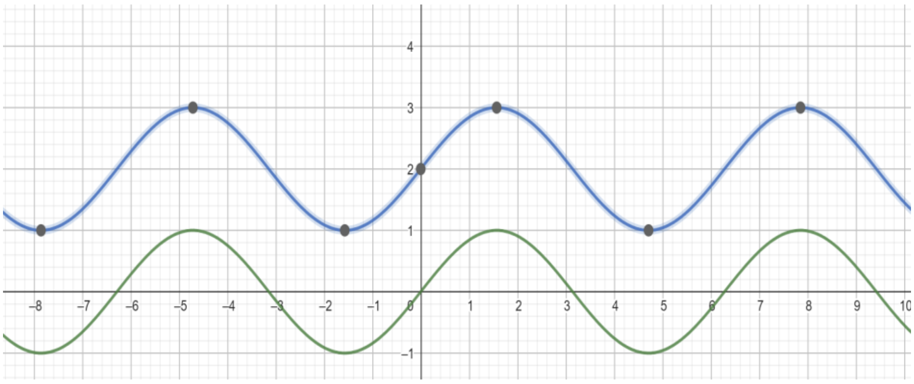
Step 2.

Draw the graphs of the functions $\sin(x)$ and $\sin(x - 1)$. Discuss how the coefficient a affects the graph of the function $f(x) = \sin(x - \mathrm{a})$.



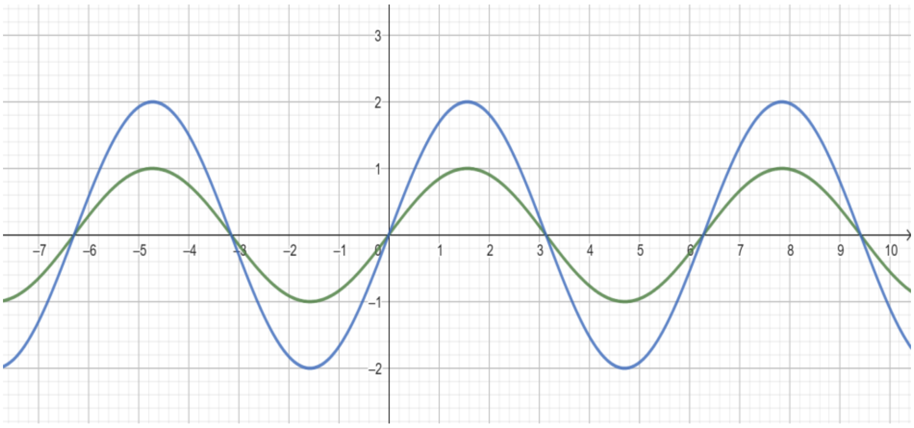
Step 3.

Draw the graphs of the functions $\sin(x)$ and $\sin(x) + 2$. Discuss how the coefficient a affects the graph of the function $f(x) = \sin(x) + a$.



Step 4.

Draw the graphs of the functions $\sin(x)$ and $2 \cdot \sin(x)$. Discuss how the coefficient a affects the graph of the function $f(x) = a \cdot \sin(x)$.



Step 5.

Repeat steps 1, 2, 3 and 4 for the function $\cos(x)$.

Step 6.

Draw the graph of the function $f(x) = 2\sin(x - 1) + 2$.

What questions should I ask so that students can share their thoughts?

How does the set of values of the function $f(x) = a\sin(x - b) + c$ change depending on the parameters a, b, c?

Lesson scenario 2

Lesson title

A vector-valued function

Learning outcomes

- Uses a vector-valued function.
- Analyzes a vector-valued function.

The course of the lesson

Step 1.

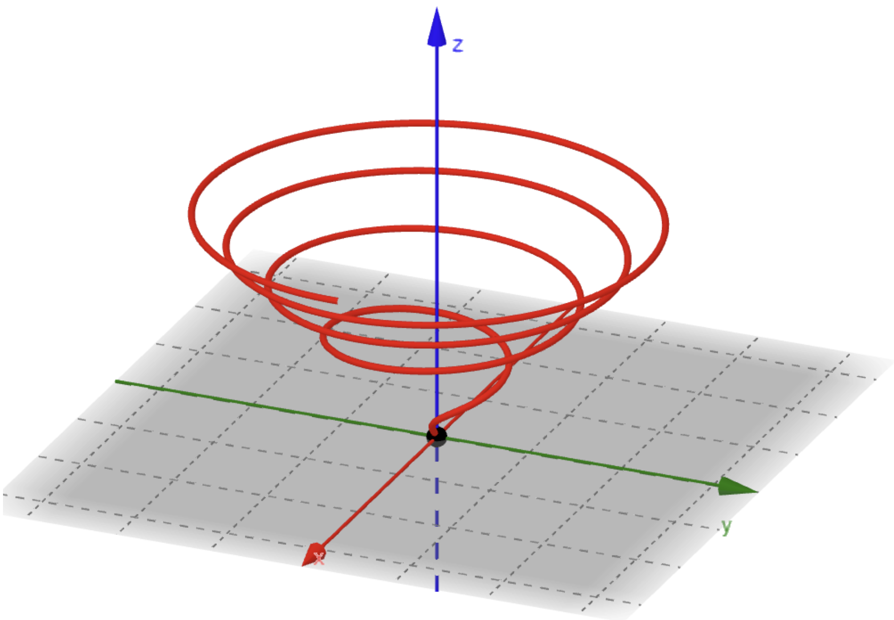
Introduction of the definition of a vector function with three coordinates.

A vector function with three coordinates is called a function of the form $f(t) = [x(t), y(t), z(t)]$, where $x(t), y(t), z(t)$ are scalar functions of the variable t.

Function examples.

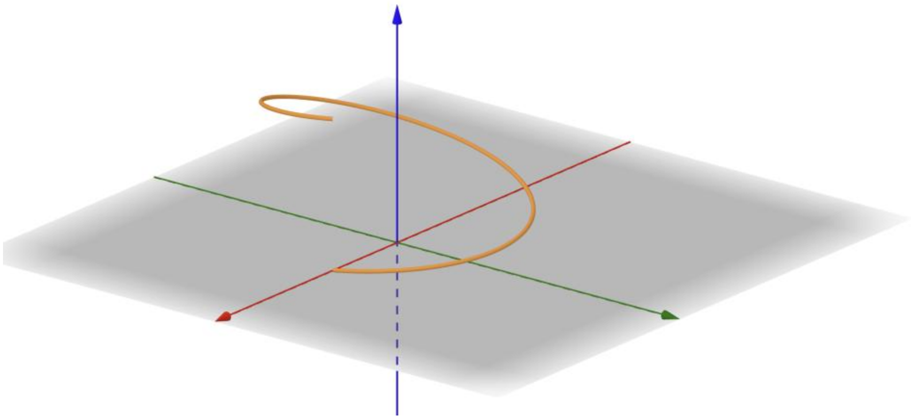
Step 2.

The function is given parametrically $[t, \sqrt{t}\sin(t), \sqrt{t}\cos(t)]$, where t is any real number.



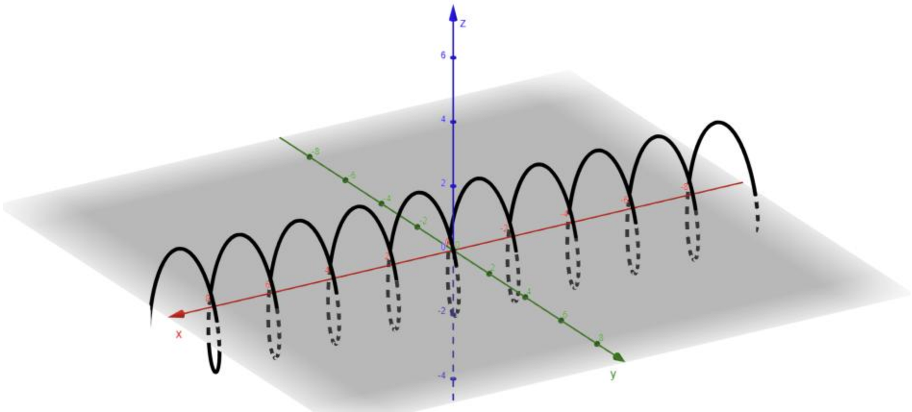
Step 3.

The function is given parametrically $[2\cos(t), 3\sin(t), t]$, where t is any real number.



Step 4.

The function is given parametrically $[t, \sin(\pi t), \cos(\pi t)]$, where t is any real number.



Step 5.

Draw the graph of the function $[t, 0, t^2]$.

Step 6.

Analyze the function $[t, a_1\sin(t) + b_1\cos(t), a_2\sin(t) + b_2\cos(t)]$.

What questions should I ask so that students can share their thoughts?

How to introduce the concept of a function whose values are n-dimensional vectors?

Suggestions and tips for teachers

1. Question for students: How to introduce the concept of the derivative of a vector function?
2. Question for students: How to introduce the concept of a vector function of two variables?

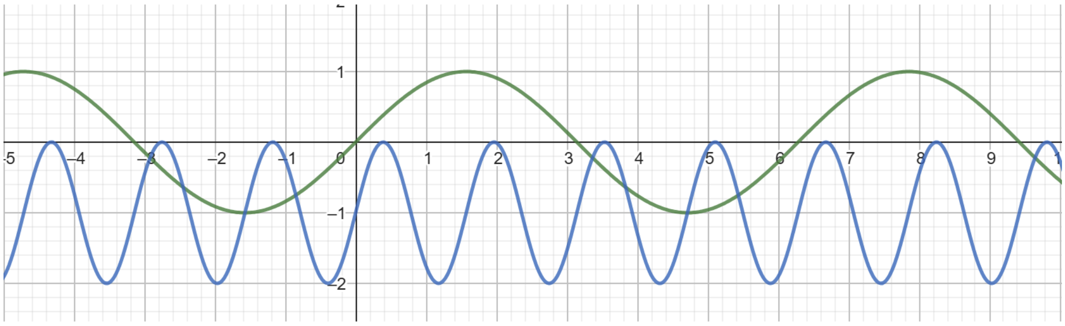
3. Introduce the concept of rotation of a vector function and show its applications.
4. You can use the WOLFRAM program to draw vector functions.

Worksheets for students

Exercise 1.

Draw the graph of the function $f(x) = \sin(4x) - 1$.

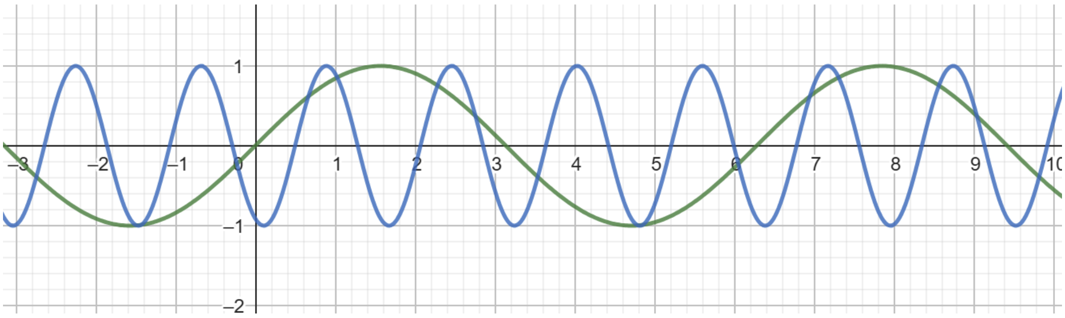
Solution



Exercise 2.

Draw the graph of the function $f(x) = \sin(4x - 2)$.

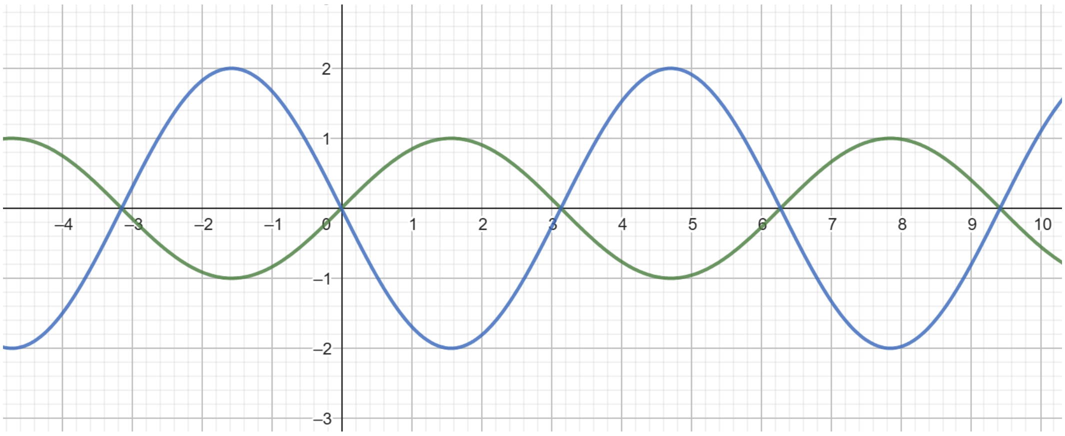
Solution



Exercise 3.

Draw the graph of the function $f(x) = -2\sin(x)$.

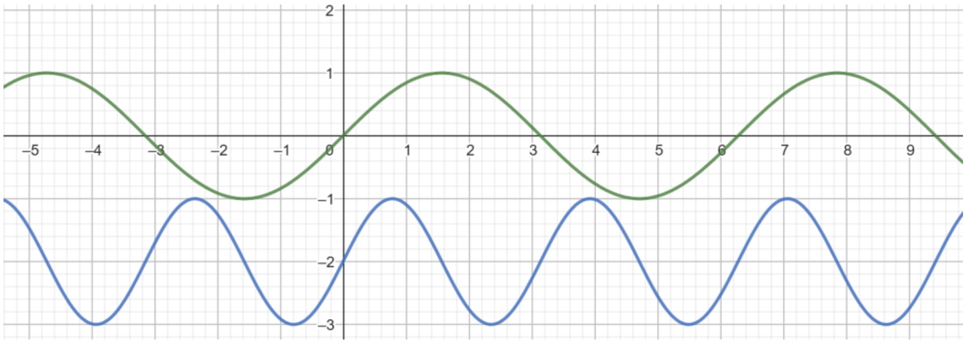
Solution



Exercise 4.

Draw the graph of the function $f(x) = -\sin(2x) + 1$.

Solution



Exercise 5.

Determine the domain of the vector function $f(t) = [\sqrt{t}, t, \frac{1}{t-2}]$.

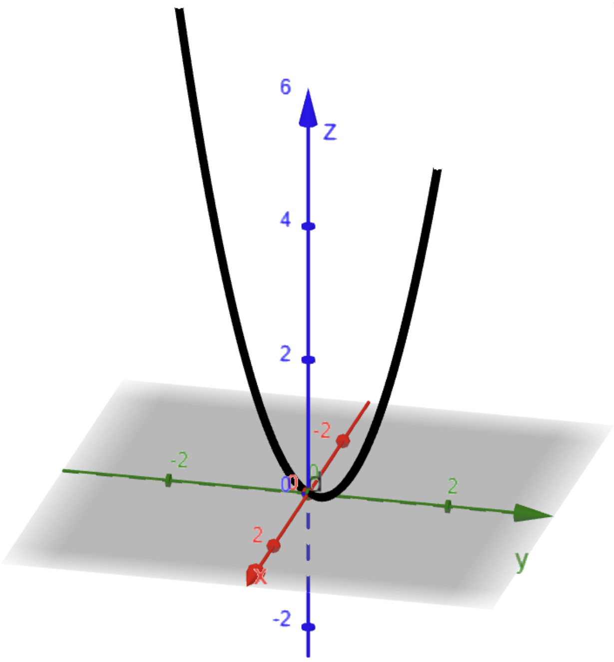
Solution

$t \in < 0,2) \cup (2, \infty)$.

Exercise 6.

Draw the graph of the vector function $f(t) = [t, t, t^2]$ for $t \in < -3,3 >$.

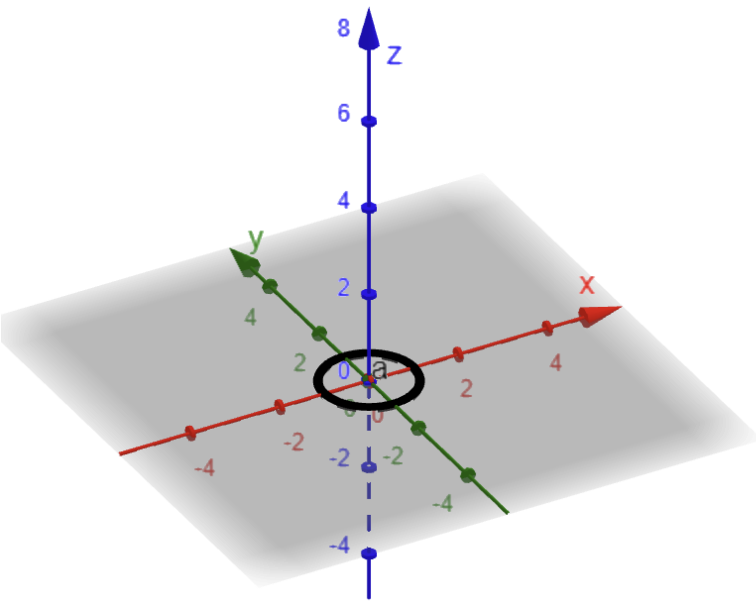
Solution



Exercise 7.

Draw the graph of the vector function $f(t) = [\sin(t), \cos(t), 0]$ for $t \in < 0,2\pi >$.

Solution



Exercise 8.

Calculate the value of the vector function $f(t) = [t, 2\sin(t) + \cos(t), \sin(t) - \cos(t)]$ for $t = 0$.

Solution

$f(t) = [0,1,-1]$.

Module 2: Angles in a cuboid

Lesson scenarios with VR applications

Lesson scenario 1

Lesson title

Angles in a cuboid

Learning outcomes

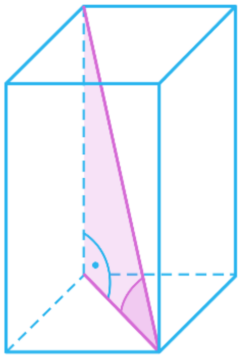
- Recognizes angles in a cuboid.

The course of the lesson

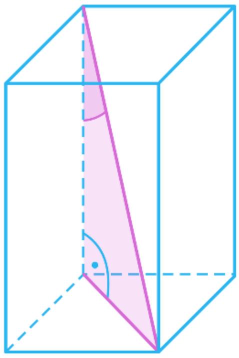
Step 1.

Introduction of definitions.

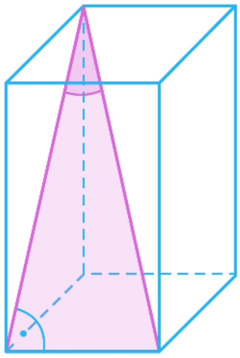
The angle between the diagonal of a cuboid and the diagonal of its base.



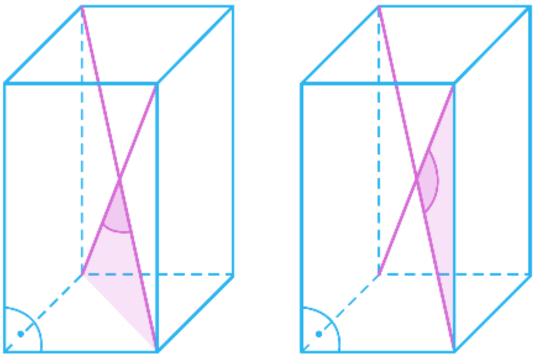
The angle between the diagonal of a cuboid and its side edge.



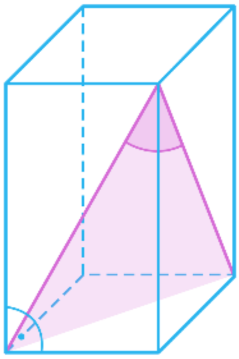
The angle between the diagonal of the cuboid and the diagonal of the side face.



The angle between the diagonals of a cuboid.

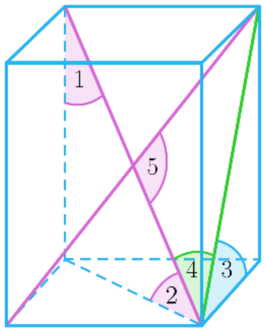


The angle between the diagonals of adjacent side walls.



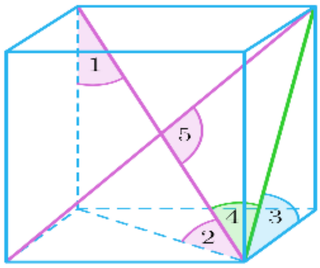
Step 2.

Name each angle: 1, 2, 3, 4 and 5.



Step 3.

Determine the measures of all angles for the cube.



What questions should I ask so that students can share their thoughts?

What is the range of angle values 1, 2, 3 and 4 in a cuboid? What is the smallest and largest angle value?

Lesson scenario 2

Lesson title

Calculation of edge length, surface area and volume in a cuboid

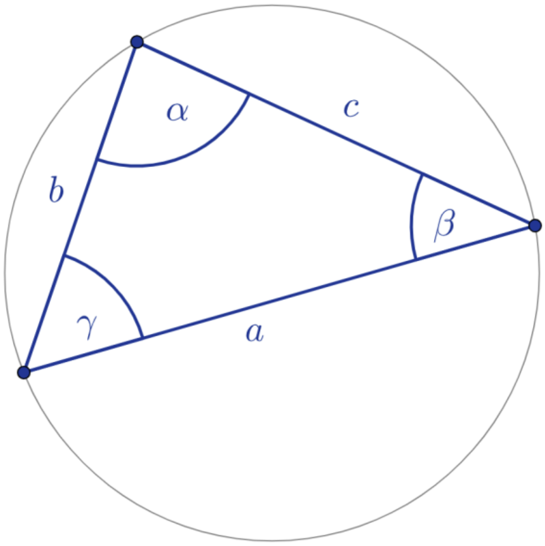
Learning outcomes

- Uses angles in a cuboid to calculate: edge length, surface area, volume.

The course of the lesson

Step 1.

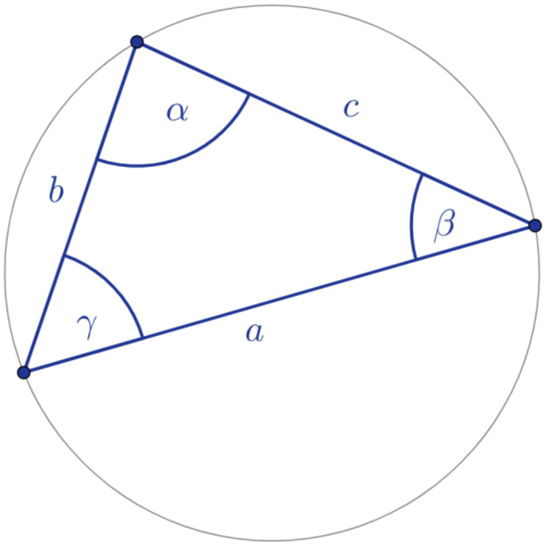
A reminder of the sine theorem.



$$\frac{a}{\sin(\alpha)} = \frac{b}{\sin(\beta)} = \frac{c}{\sin(\gamma)}$$

Step 2.

A reminder of the cosine theorem.



$$a^2 = b^2 + c^2 - 2ab\cos(\alpha)$$

Complete the remaining dependencies:

$$b^2 = a^2 + \dots.^2 - 2 \dots \cos(\dots)$$

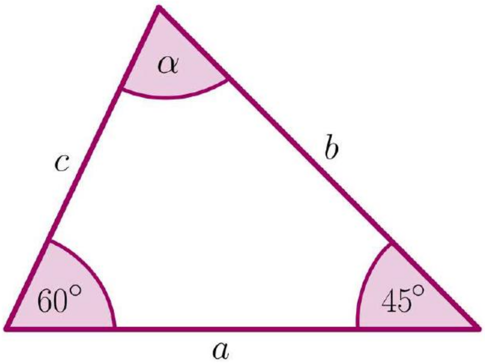
$$c^2 = a^2 + \dots.^2 - 2 \dots \cos(\dots)$$

Step 3.

Calculations using the sine and cosine theorems.

Exercise 1.

Calculate the length of segment b if $c = 5$.

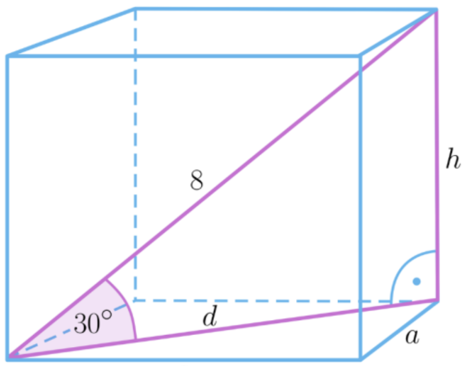


Step 4.

Calculations of edge length, surface area and volume in a cuboid.

Exercise 2.

Calculate the total surface area and volume of the cuboid if the base is square.



What questions should I ask so that students can share their thoughts?

In what situations is the sine theorem and in what situations the cosine theorem used?

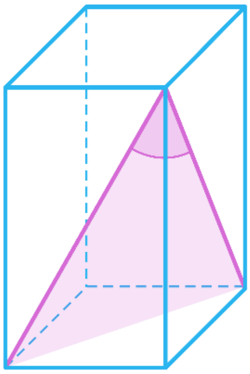
Suggestions and tips for teachers

1. Question for students: Is it possible to prove the Pythagorean theorem using the cosine theorem?
2. Question for students as their own work: How are angles in prisms used in architecture?
3. Is it possible to construct a prism in which all angles between the side edges and the bases are equal? Justify.
4. You can use the WOLFRAM program for trigonometric calculations.

Worksheets for students

Exercise 1.

Give the name of the angle.



Solution

The angle between the diagonals of the side faces of a cuboid.

Exercise 2.

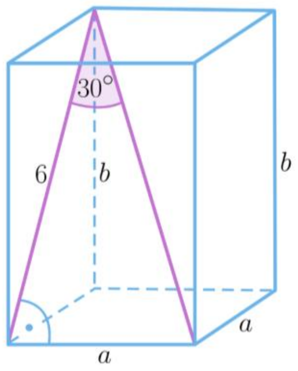
Calculate the volume of the cuboid for $\alpha = 45°$.

Solution

$V = 48\sqrt{2}$.

Exercise 3.

The volume of the cuboid shown in the figure is $48\sqrt{3}$. Calculate the dimensions of the cuboid.

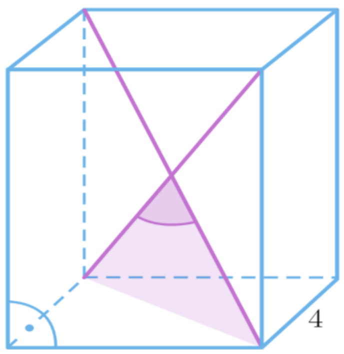


Solution

$a = 2\sqrt{6}, b = 2\sqrt{3}.$

Exercise 4.

The length of the side of the cube is 4. Determine the cosine measure of the angle marked in the figure.



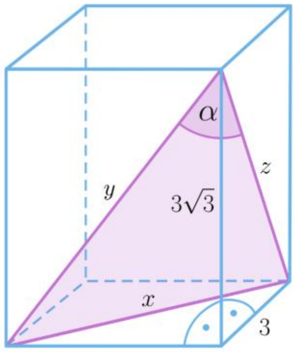
Solution

$\cos(\alpha) = \frac{-2}{3}.$

Exercise 5.

The base of the cuboid shown in the figure is 3 by 3 and the side edge is $3\sqrt{3}$. Determine the measure of the angle between the diagonals of adjacent side faces of the cuboid.

Solution

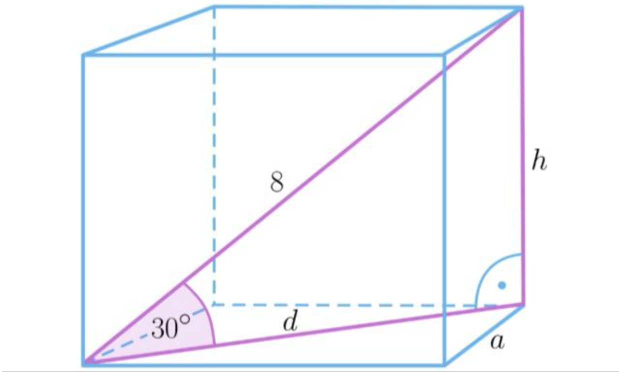


Note that $y = x$ and $x = 3\sqrt{2}$. Using the cosine theorem for a triangle with sides x, y, z we get $\cos(\alpha) = \frac{45}{72}$.

Exercise 6.

The diagonal of a cuboid with a square base has length 8 and is inclined to the plane of the base at an angle of 30°. Calculate the dimensions of the edges of the cuboid.

Solution

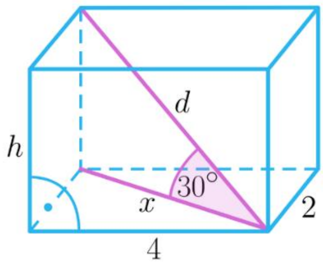


$\frac{h}{8} = \sin(30°) = \frac{1}{2}$, therefore $h = 4$ and $d = 4\sqrt{3}$. Therefore $a = 2\sqrt{6}$.

Exercise 7.

The edge lengths of the cuboid are $4, 2, h$. Determine the length of the diagonal of the cuboid if the angle between the diagonal of the cuboid and the diagonal of the base of the cuboid is 30°.

Solution

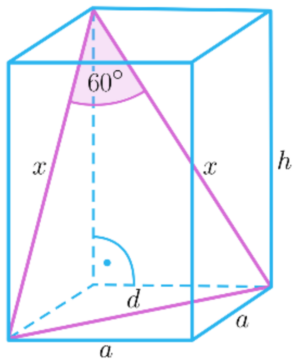


Using the Pythagorean theorem determine $x = 2\sqrt{5}$. Since $\frac{x}{d} = \cos(30°)$, so $d = \frac{4}{3}\sqrt{15}$.

Exercise 8.

A cuboid with a square base is given. Prove that a cuboid is a cube if the angle between the diagonals of adjacent faces is 60°.

Solution



The triangle x, x, d is equilateral, so $x = a\sqrt{2}$. Using the Pythagorean theorem for a triangle with sides x, a, h, we get that $h = a$.

Module 3: Angles in a pyramid

Lesson scenarios with VR applications

Lesson scenario 1

Lesson title

Pyramids and total surface area

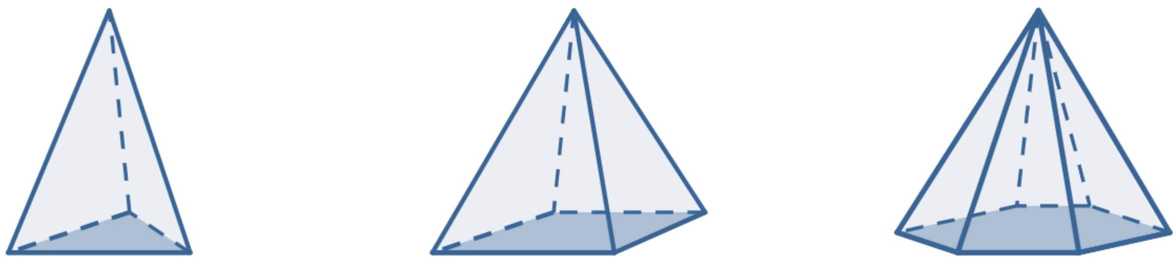
Learning outcomes

- Determines the volume and total surface area of the pyramid.

The course of the lesson

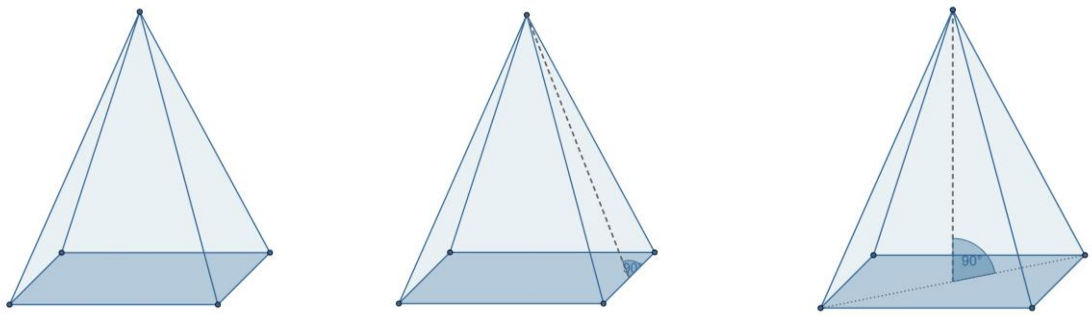
Step 1.

Introduction of the definition of basic pyramids divided according to the base.



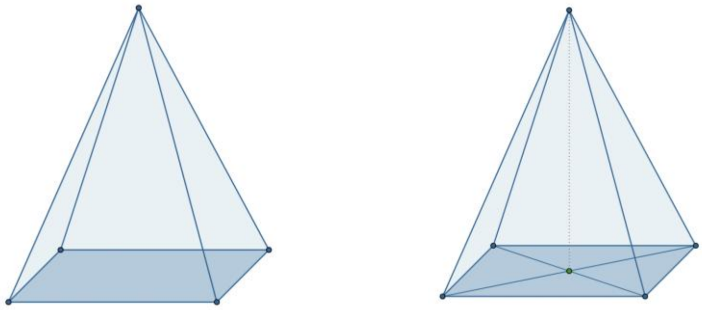
Step 2.

Drawing the height of the side wall and the height of the pyramid.



Step 3.

Drawing the diagonals of the base of a pyramid.



Step 4.

Discussion of formulas for the total surface area of a pyramid.

What questions should I ask so that students can share their thoughts?

What is the ratio of the volume of a prism to the volume of a pyramid with the same bases and equal heights?

Lesson scenario 2

Lesson title

Angles in a pyramid

Learning outcomes

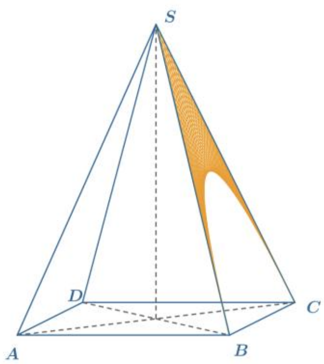
- Recognizes angles in a pyramid.

The course of the lesson

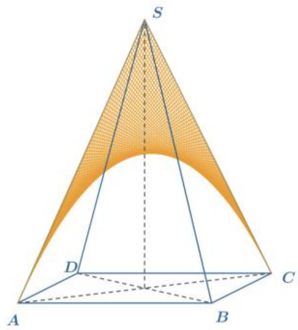
Step 1.

Introduction of the definition of angles between edges in a pyramid.

The angle between the side edges.



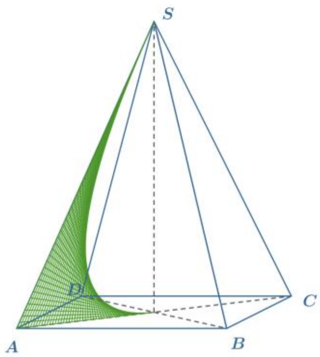
The angle between opposite side edges.



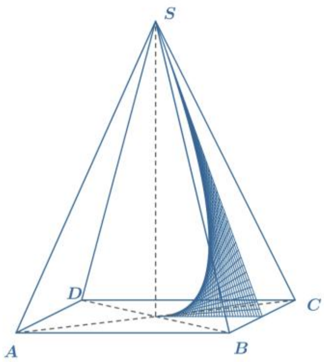
Step 2.

Introducing the definition of the remaining angles in the pyramid.

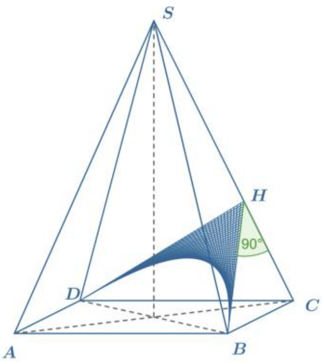
The angle between the edge of a side wall and the diagonal of the base..



The angle of inclination of the side wall to the base.



The angle between adjacent side walls.



Step 3.

Determining the values of all angles in a tetrahedron.

What questions should I ask so that students can share their thoughts?

How to determine the radius of a sphere circumscribed by a tetrahedron?

Suggestions and tips for teachers

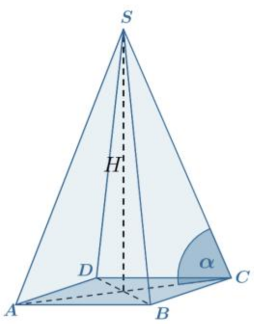
1. Suggestion for students: a pyramid is called straight if its side edges are equal.
2. Suggestion for students: a pyramid is called a tetrahedron if all its side faces are equilateral triangles.
3. Question for students: Draw nets of prisms.
4. You can use the WOLFRAM program to calculate the volume of pyramids.

Worksheets for students

Exercise 1.

A regular quadrangular pyramid is given. The length of the base edge is 6 and the height is $3\sqrt{2}$. Calculate the angle between the side edge and the base plane.

Solution

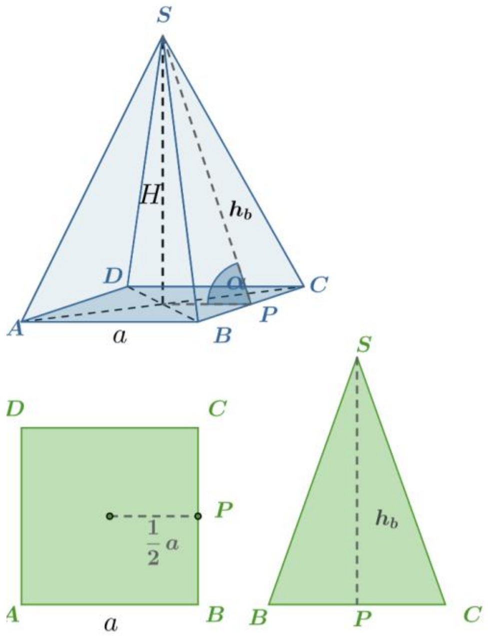


$|\overline{AC}| = 6\sqrt{2}$. $\tan(\alpha) = \frac{H}{\frac{|AC|}{2}} = 1$. Therefore $\alpha = 45°$.

Exercise 2.

A regular quadrangular pyramid is given. The length of the base edge is 6 and the height is $3\sqrt{2}$. Calculate the tangent of the angle between the side wall and the base plane.

Solution

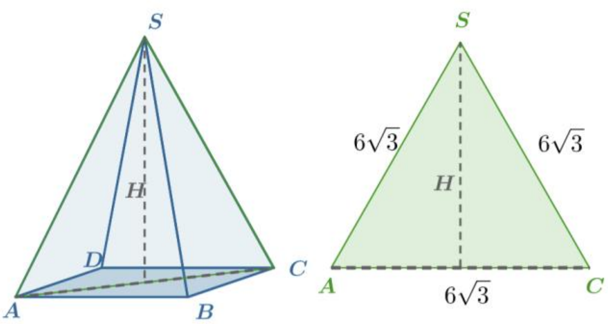


$\tan(\alpha) = \frac{H}{\frac{1}{2}a} = \frac{3\sqrt{2}}{3} = \sqrt{2}.$

Exercise 3.

A regular quadrangular pyramid is given. The length of the base edge is $3\sqrt{6}$ and the length of the side edges is $6\sqrt{3}$. Calculate the height of the pyramid.

Solution

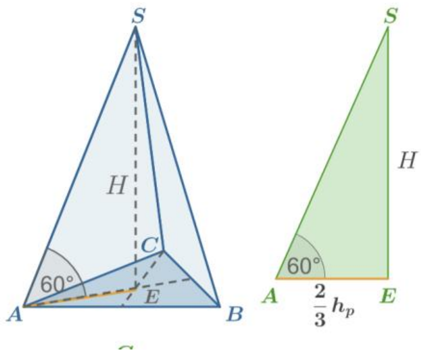


$H = 9.$

Exercise 4.

A regular triangular pyramid is given. The length of the base edge is $a = 3\sqrt{3}$ and the angle between the side edge and the base plane is $60°$. Calculate the height of the pyramid.

Solution



$h_p = \frac{a\sqrt{3}}{2} = \frac{9}{2}$. Therefore $H = \frac{2}{3}h_p \tan(60°) = 3\sqrt{3}$.

Exercise 5.

Calculate the height of a regular tetrahedron with edge length 3.

Solution

$\sqrt{6}$.

Exercise 6.

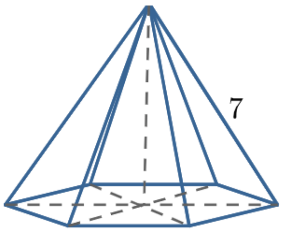
The three faces of the pyramid are isosceles right triangles with a side length of 1. Calculate the area of the fourth face.

Solution

$\frac{\sqrt{3}}{2}$.

Exercise 7.

The base area of the right hexagonal pyramid shown in the figure is $\frac{75\sqrt{3}}{2}$. Calculate the cosine of the angle of inclination of the side edge to the base plane.

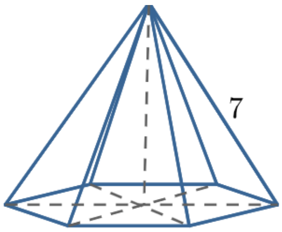


Solution

$P = \frac{75\sqrt{3}}{2} = 6\frac{a^2\sqrt{3}}{4}$. Therefore $a = 5$ and $\sin(\alpha) = \frac{5}{7}$.

Exercise 8.

The base area of the right hexagonal pyramid shown in the figure is $\frac{75\sqrt{3}}{2}$. Calculate the height of this pyramid.



Solution

$P = \frac{75\sqrt{3}}{2} = 6\frac{a^2\sqrt{3}}{4}$, so $a = 5$. From the Pythagorean theorem is $H^2 = 7^2 - 5^2$, therefore $H = 2\sqrt{6}$.

Module 4: Non-Euclidean geometry

Lesson scenarios with VR applications

Lesson scenario 1

Lesson title

Euclidean geometry

Learning outcomes

- Knows the axioms of Euclidean geometry.

The course of the lesson

Step 1.

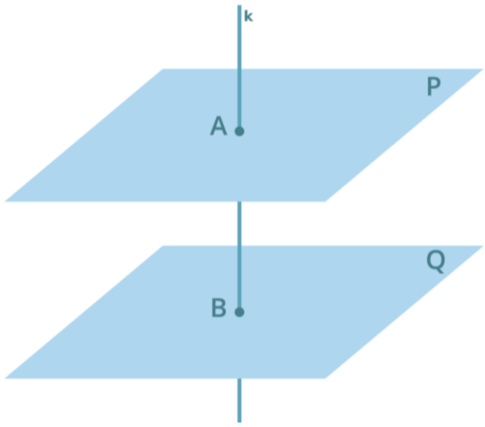
Introduction of primary concepts in geometry: point, line, plane.

Step 2.

Learning the axioms of Euclidean geometry.

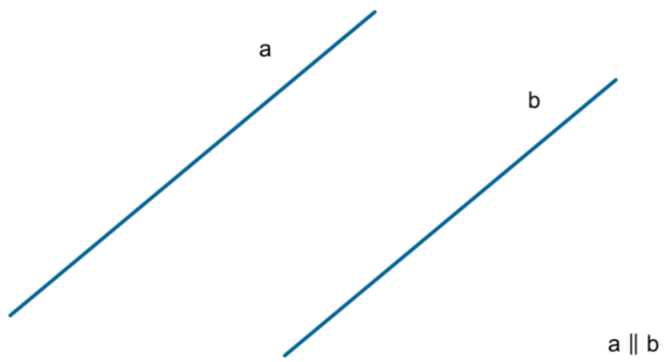
Step 3.

Discussion of the position of planes in three-dimensional space, e.g. parallel planes.



Step 4.

Discussing the position of two lines in a plane, e.g. parallel lines.



Step 5.

Discussing the position of a line and a point on a plane and space.

Step 6.

Discussing the position of a plane and a point in space.

What questions should I ask so that students can share their thoughts?

What is the position of two lines in three-dimensional space?

Lesson scenario 2

Lesson title

Basics of non-Euclidean geometry

Learning outcomes

- Uses non-Euclidean geometry.

The course of the lesson

Step 1.

Introduction to non-Euclidean geometry, historical elements.

Step 2.

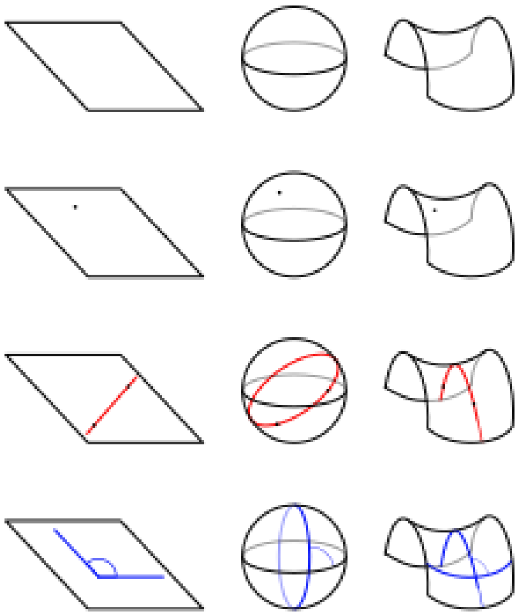
Discussion of the assumptions of elliptic geometry.

Step 3.

Discussion of the assumptions of hyperbolic geometry.

Step 4.

Plane, point, line, angle in terms of Euclidean, spherical and hyperbolic geometry.



https://pl.wikipedia.org/wiki/Geometria_nieeuklidesowa

What questions should I ask so that students can share their thoughts?

In hyperbolic geometry, can the sum of the angles in a triangle be less than 180°?

Suggestions and tips for teachers

1. Question for students: How does non-Euclidean geometry differ from Euclidean geometry?
2. Question for students: What does a circle look like in non-Euclidean geometry?
3. Mathematicians who contributed to the development of non-Euclidean geometry.
4. Considerations on the use of non-Euclidean geometry.

Worksheets for students

Exercise 1.

How many lines parallel to a given line can be drawn through a given point?

Solution

Only one line parallel to a given line can be drawn through a given point.

Exercise 2.

How many planes parallel to a given plane can be drawn through a given point?

Solution

Only one plane parallel to a given plane can be drawn through a given point.

Exercise 3.

How many points clearly define the plane?

Solution

Three non-collinear points.

Exercise 4.

How to clearly determine a plane using two lines?

Solution

For example, using two lines intersecting at one point.

Exercise 5.

What can be a common part of a line and a plane?

Solution

The common part of a line and a plane may be: an empty set, a point, a line.

Module 5: Maxima and minima of functions

Lesson scenarios with VR applications

Lesson scenario 1

Lesson title

Local minimum and maximum: definition, geometric interpretation

Learning outcomes

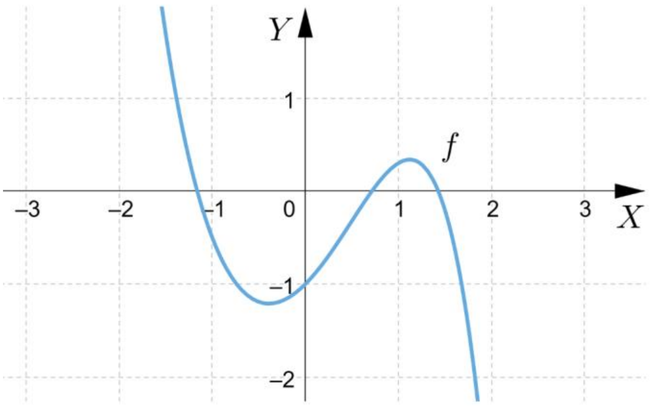
- Reads local extrema of functions of two variables.
- Indicates the differences between the extremum and the global function of two variables.

The course of the lesson

Step 1.

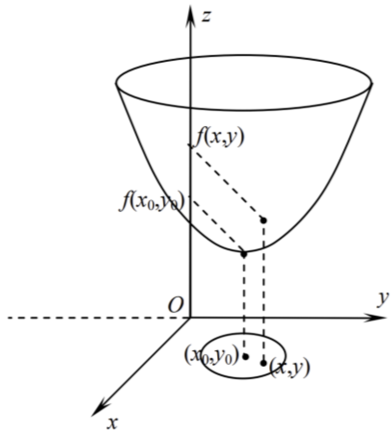
Reminder of the definition of the minimum and maximum of a function of one variable.

Indication of the local minimum and maximum of a function of one variable.



Step 2.

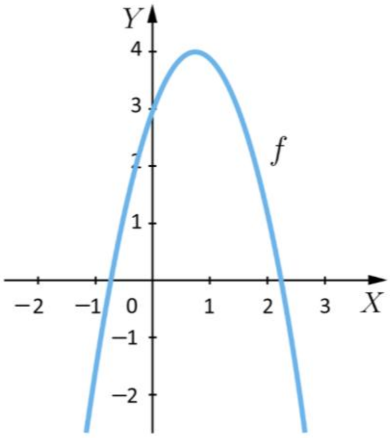
Formulating the definition of the local minimum of a function of two variables based on the graph.



Step 3.

What is the difference between local minimum and global minimum?

Give an example. Justification based on the graph for the functions f and $x \in [-1,3]$.



Step 4.

Formulating the definition of the global minimum of a function of two variables.

What questions should I ask so that students can share their thoughts?

Can a function have a minimum and a zero of function at a fixed point?

Lesson scenario 2

Lesson title

Necessary and sufficient conditions for an extremum of a function of two variables

Learning outcomes

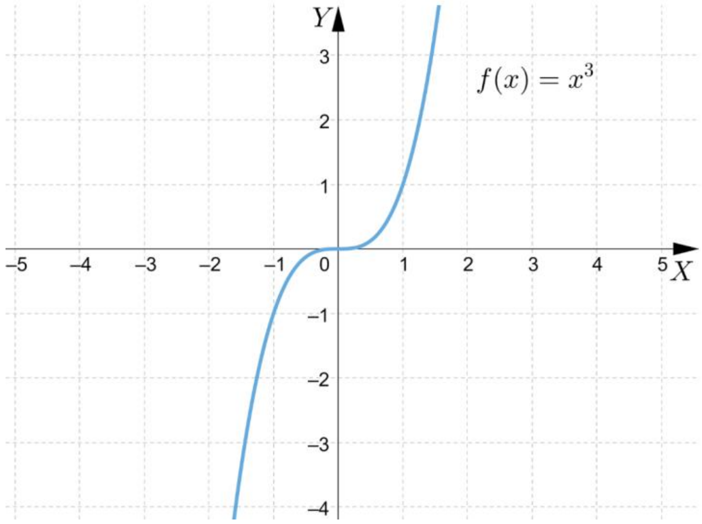
- Uses the necessary and sufficient conditions for the extremum of the function of two variables.

The course of the lesson

Step 1.

Reminder of the necessary condition for an extremum of a function of one variable.

Does the function $y = x^3$ for $x = 0$ meets the necessary condition for the extremum of the function?



Step 2.

Reminder of the sufficient condition for the extremum of a function of one variable.

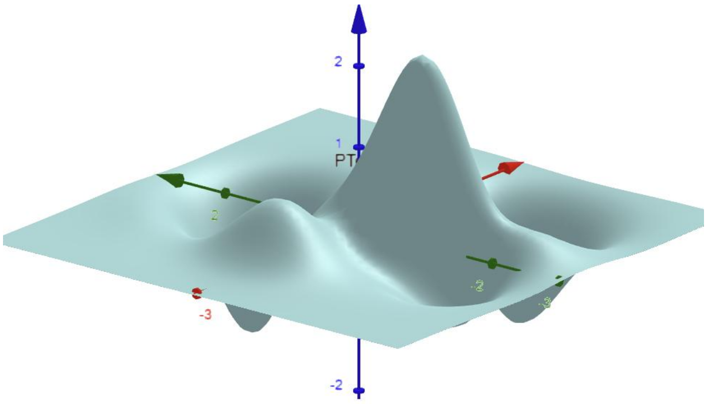
Step 3.

Formulating the necessary condition for the extremum of a function of two variables.

Step 4.

Formulating the sufficient condition for the extremum of a function of two variables.

Indication of the points at which the function meets the necessary and sufficient conditions for the extremum of a function of two variables



Step 5.

Comparison of the necessary and sufficient conditions for the extremum of functions of one and two variables.

What questions should I ask so that students can share their thoughts?

Is there a function in which first-order partial derivatives do not exist and the function has a local extremum at this point?

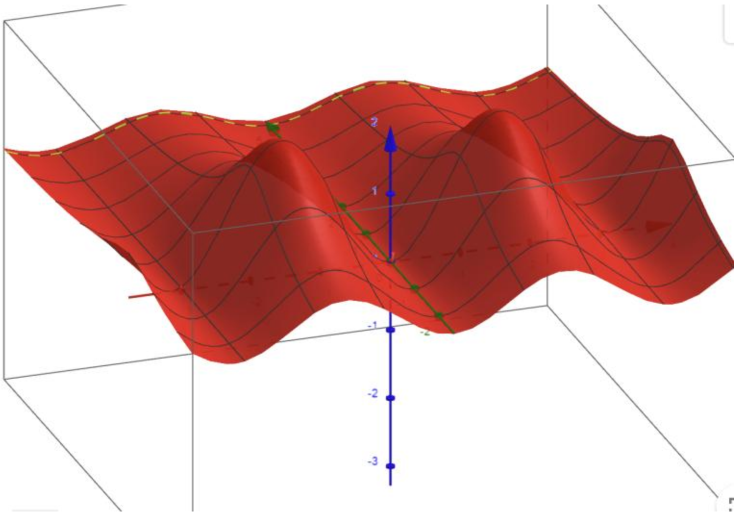
Suggestions and tips for teachers

1. Question for students: Does the function have no extremum due to the fact that there are no partial derivatives?
2. Show the student examples of functions of two variables for which the necessary condition for reaching the extreme of the function is met, but the sufficient condition is not met.
3. Question for students: Formulate the necessary condition for the extremum of a function for three variables.
4. You can use the WOLFRAM program to find the maximum and minimum of functions of two variables.

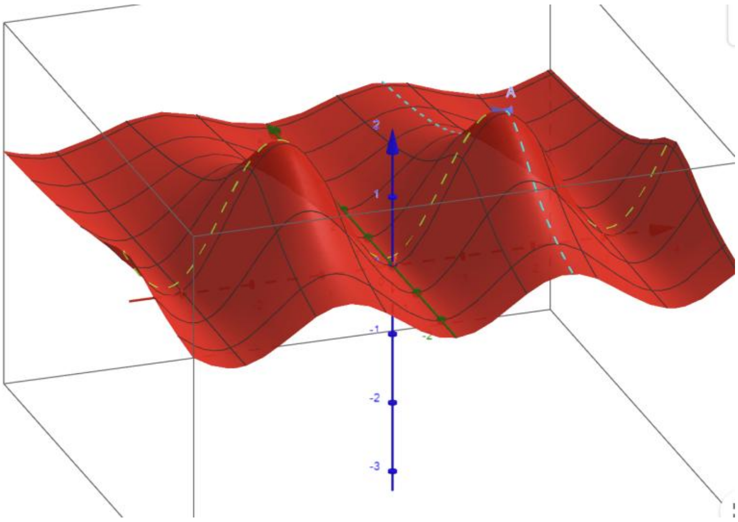
Worksheets for students

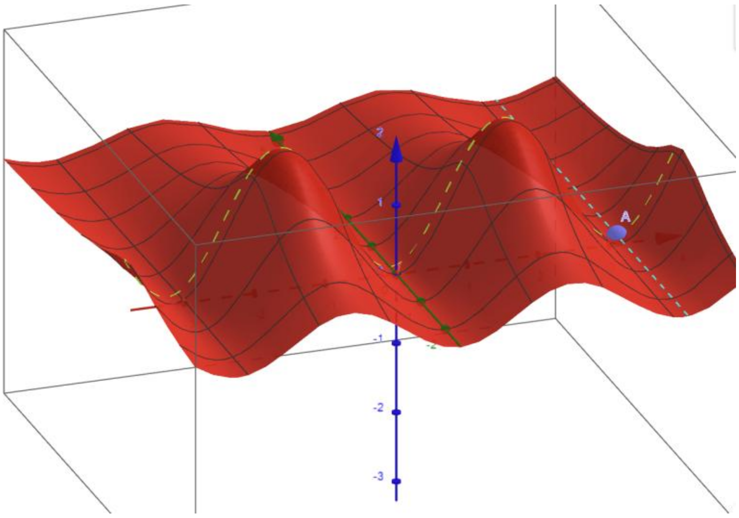
Exercise 1.

Indicate an example local minimum and local maximum of the function shown in the graph.



Solution





Exercise 2.

A top-open container in the shape of a cuboid is given. Its volume is 256. Determine the dimensions of this container so that the surface area of the cuboid is the smallest.

Solution

$a = 8, b = 8, h = 4.$

Exercise 3.

Give an example of a function of two variables that has infinitely many local extrema.

Solution

$f(x, y) = \sin(x) + \sin(y).$

Exercise 4.

Determine local extrema of function $f(x, y) = x^2y + y^3 + 6xy$.

Solution

$\min = -6\sqrt{3}.\ \max = 6\sqrt{3}.$

Exercise 5.

Determine local extrema of function $f(x, y) = x^3 + y^3 - 6xy$.

Solution

$\min = -8.$

Exercise 6.

Is the function $f(x, y) = x^2y + y^3 + 6xy$ at point $P(0,0)$ has a local extremum?

Solution

No.

Exercise 7.

Determine local extrema of function $f(x, y) = e^{x^2+y^2}(x^2 + y^2)$.

Solution

$\min = 0.$

Exercise 8.

Is the function $f(x, y) = |x| + |y|$ at point $P(0,0)$ has a local minimum?

Solution

Yes.

Module 6: Systems of linear equations

Lesson scenarios with VR applications

Lesson scenario 1

Lesson title

Geometric interpretation of linear equations in spaces

Learning outcomes

- Visualize different possibilities of the solution set of a system of linear equations.
- Recognize different possibilities of the solution set of a system of linear equations.

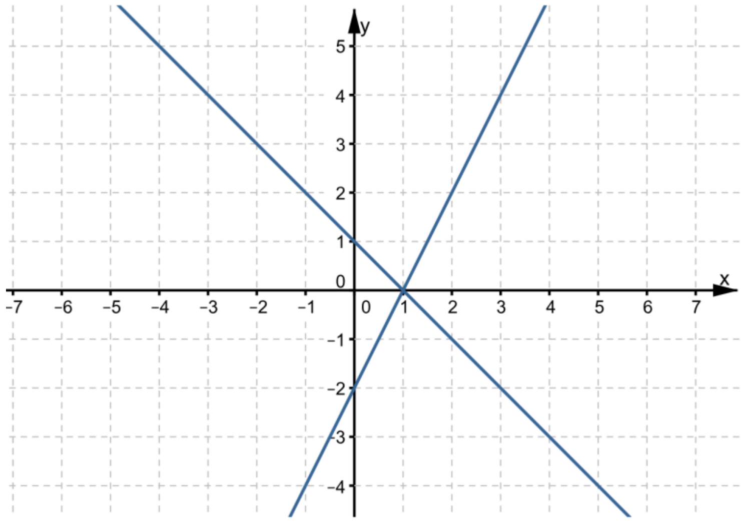
The course of the lesson

Step 1.

Reminder of the interpretation of a system of linear equations for two variables.

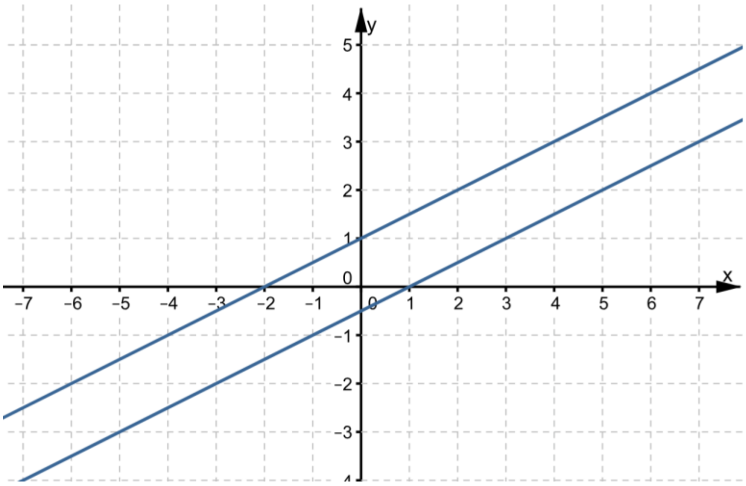
Step 2.

Presentation of the solution of a system of linear equations on a graph.



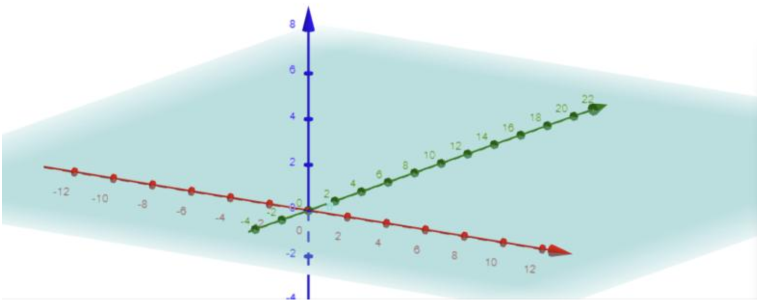
Step 3.

Presentation of the solution of a system of linear equations on a graph.

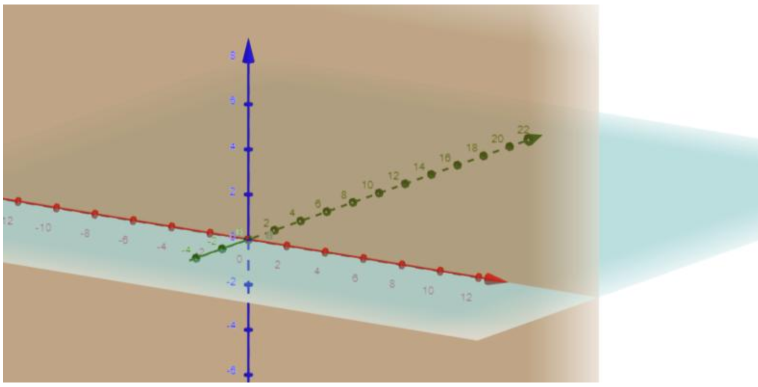


Step 4.

Draw a plane $x = 0$.



Draw the $x = 0$ and $y = 0$ planes.



What questions should I ask so that students can share their thoughts?

What are the possible solutions to systems of linear equations in four-dimensional space?

Lesson scenario 2

Lesson title

Solving linear equations

Learning outcomes

- Solves systems of linear equations in three-dimensional space.

The course of the lesson

Step 1.

Solving a system of linear equations.

$$\begin{cases} x + y + z = 2 \\ x + y + z = 2 \end{cases}$$

Step 2.

Solving a system of linear equations.

$$\begin{cases} x + y + z = 2 \\ x + y + z = 4 \end{cases}$$

Step 3.

Solving a system of linear equations.

$$\begin{cases} x + y + z = 2 \\ x + y + 2z = 2 \end{cases}$$

Step 4.

Solving a system of linear equations.

$$\begin{cases} x + y + z = 2 \\ 0x + 0y + 0z = 0 \end{cases}$$

What questions should I ask so that students can share their thoughts?

Can the entire space be the solution to a system of linear equations?

Suggestions and tips for teachers

5. Learn about different methods for solving systems of linear equations.
6. Before starting to solve a system of linear equations, it is worth rearranging the equations by moving all terms with unknowns to the left side and free terms to the right.
7. Check the solution to the system of linear equations, e.g. by substituting the found solution.
8. Systems of linear equations are used in artificial intelligence.
9. You can use the WOLFRAM program to solve systems of linear equations.

Worksheets for students

Exercise 1.

Give values of parameters a, b so that the solution to the system of linear equations is an empty set.

$$\begin{cases} 2x + y + 3z = 2 \\ 2x + y + az = b \end{cases}$$

Solution

$a = 3, b \neq 2.$

Exercise 2.

Give values of parameters a, b so that the solution to the system of linear equations is a line.

$$\begin{cases} 2x + y + 3z = 2 \\ 2x + y + az = b \end{cases}$$

Solution

$a \neq 3, b \in R.$

Exercise 3.

Give values of parameters a, b so that the solution to the system of linear equations is a point.

$$\begin{cases} 2x + y + 3z = 2 \\ 2x + y + az = b \end{cases}$$

Solution

$a = 3, b = 2.$

Exercise 4.

Give values of parameters a, b so that the solution to the system of linear equations is a plane.

$$\begin{cases} 2x + y + 3z = 2 \\ 0x + 0y + az = b \\ 0x + y + z = 2 \end{cases}$$

Solution

$a \neq 0, b \in R.$

Exercise 5.

Solve the system of linear equations.

$$\begin{cases} x + y + 2z = 2 \\ 2x + 2y + 4z = 6 \end{cases}$$

Solution

$\emptyset$.

Exercise 6.

Check if the system is Cramer's.

$$\begin{cases} x + y + z = 4 \\ x + y + 3z = 8 \\ x + 2y + z = 5 \\ 3x + 4y + 5z = 17 \end{cases}$$

Solution

Yes.

Exercise 7.

Solve the system of linear equations.

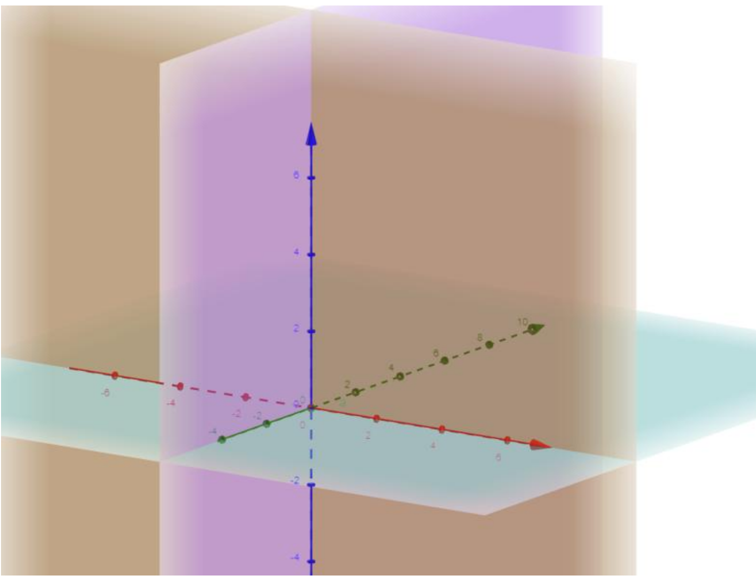
$$\{2x + y + 3z = 1$$

Solution

$y = -2x - 3z + 1$ or $[x, -2x - 3z + 1, z]$ where $x, z \in R$.

Exercise 8.

Write the system of linear equations based on the graph.



Solution

$$\begin{cases} x = 0 \\ y = 0 \\ z = 0 \end{cases}$$

Module 7: Prisms - sections of prisms, grids with prisms

Lesson scenarios with VR applications

Lesson scenario 1

Lesson title

Grids of prisms

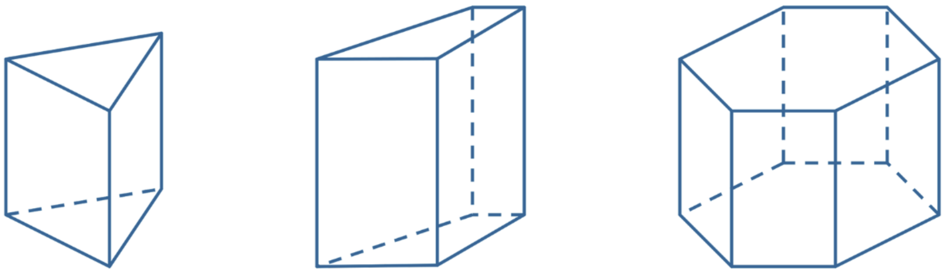
Learning outcomes

- Recognizes grids of prisms.

The course of the lesson

Step 1.

Drawing grids of prisms.



Step 2.

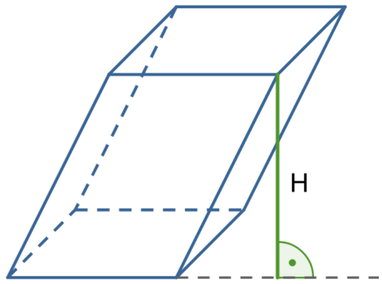
Describing what shapes grids of prisms consist of.

Step 3.

Are grids of prisms shown in the figures? Justification.

Step 4.

Drawing the grid of the prism shown in the figure.



What questions should I ask so that students can share their thoughts?

Does each given prism have only one grid?

Lesson scenario 2

Lesson title

Calculating the total surface area and volume of prisms

Learning outcomes

- Calculate the total surface area and volume of prism.

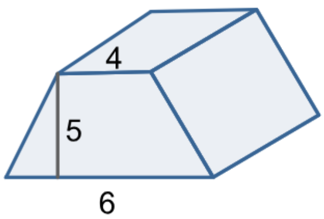
The course of the lesson

Step 1.

Reminder of formulas for the surface areas of flat figures: triangle, parallelogram, trapezoid, regular hexagon.

Step 2.

The volume of the prism shown in the figure is 500. Calculating the height of the figure.



Step 3.

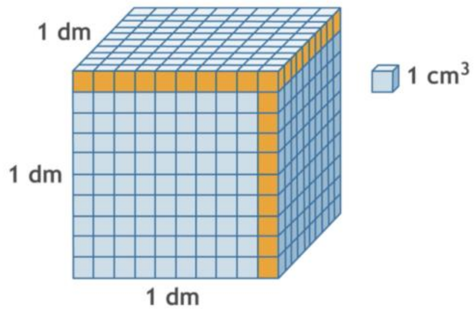
Determine the ratio of the lengths of the heights of the prisms shown in the figure so that their volumes are equal.

How will the volume of a regular hexagonal prism change if

- the length of the base edge be doubled?
- the length of the height be doubled?

Step 4.

Conversion of volume units. How much does 1 dm^3 have in cm^3?



Step 5.

How much does 1 m^3 have in dm^3?
How much does 1 m^3 have in cm^3?
How much does 1 km^3 have in cm^3?

What questions should I ask so that students can share their thoughts?

Is it possible to build different regular quadrangular prisms with the same bases, equal heights and equal volumes?

Suggestions and tips for teachers

1. Question for students: How will the volume of a prism change if the height is tripled?
2. Question for students: How will the volume of a regular quadrangular prism change if the height is tripled?

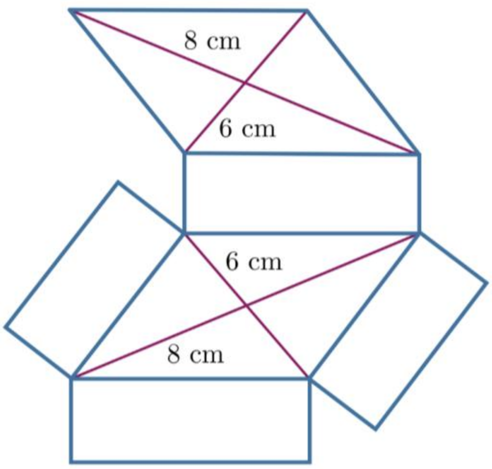
3. Question for students: How will the length of the diagonal of a regular quadrangular prism change if the height is tripled?
4. Question for students: How will the total surface area of a regular quadrangular prism change if the height is tripled?
5. You can use the WOLFRAM program to calculate the volume and total surface area of the prisms.

Worksheets for students

Exercise 1.

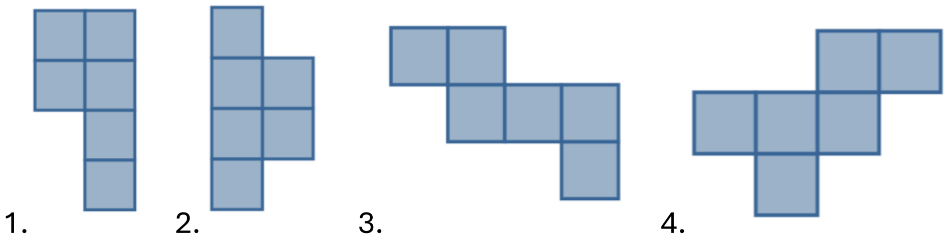
Draw the grid of a prism whose bases are rhombuses with diagonals of 6 cm and 8 cm.

Solution



Exercise 2.

Which figures show prism grids?

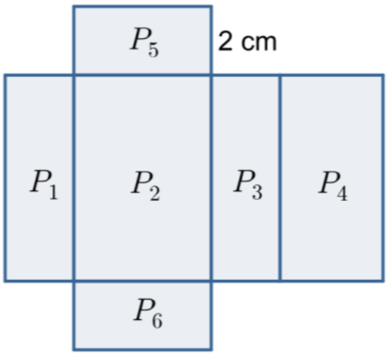


Solution

1. No.
2. No.
3. Yes.
4. Yes.

Exercise 3.

The volume of the prism shown in the figure is 24 and the total surface area is 52. Determine the lengths of the remaining edges.

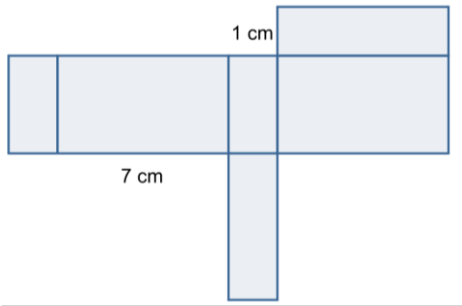


Solution

$a = 2, b = 3, c = 4.$

Exercise 4.

The total surface area of the prism shown in the figure is 78. Determine the volume of the prism.

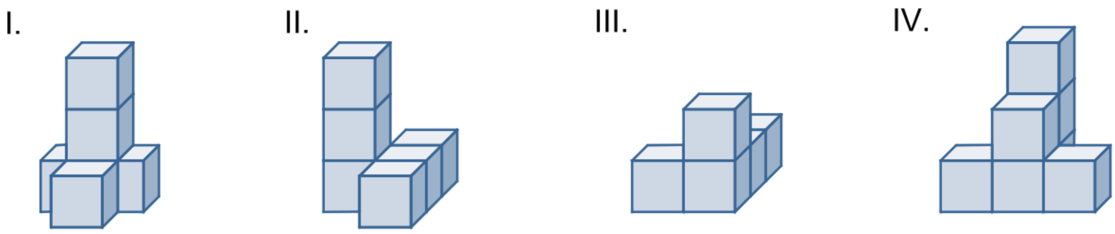


Solution

$V = 28.$

Exercise 5.

Which figure has the largest volume?

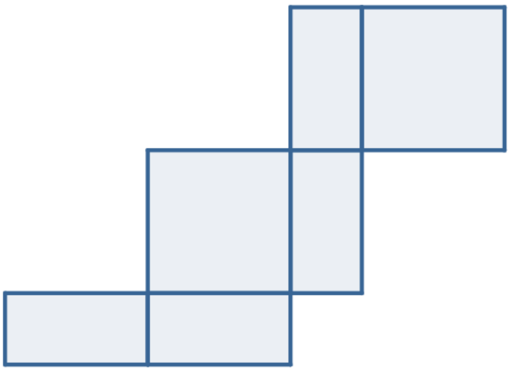


Solution

IV.

Exercise 6.

Does the figure show a prism grid?

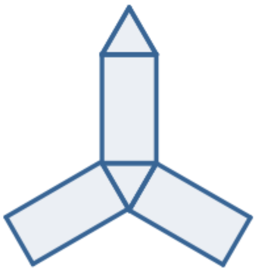


Solution

Yes.

Exercise 7.

What type of prism is shown in the figure?

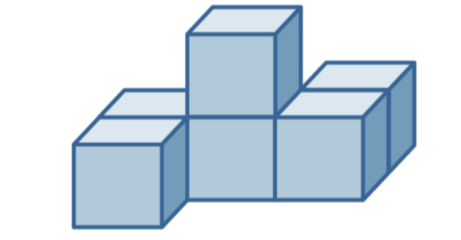


Solution

A regular triangular prism.

Exercise 8.

Draw the grid of the figure shown in the figure.



Solution

After drawing the grid, cut it out and try to assemble it.

Module 8: Pyramids – sections of pyramids, grids with pyramids

Lesson scenarios with VR applications

Lesson scenario 1

Lesson title

Grids of pyramids

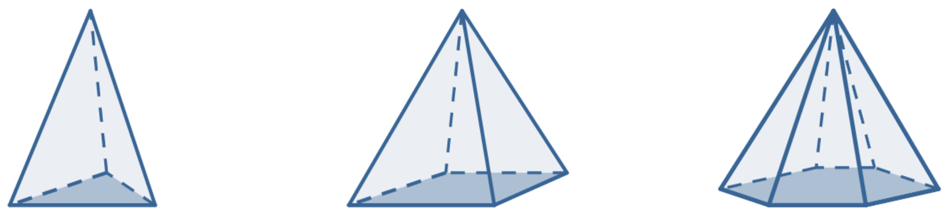
Learning outcomes

- Recognizes grids of pyramids.

The course of the lesson

Step 1.

Drawing grids of pyramids.

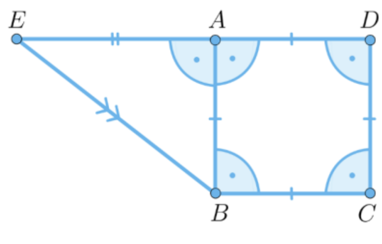


Step 2.

Describing what shapes grids of pyramids consist of.

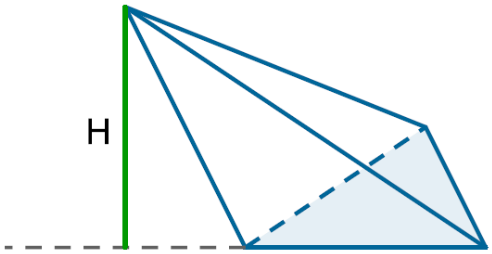
Step 3.

Drawing a pyramid grid.



Step 4.

Drawing the grid of the pyramid shown in the figure.



What questions should I ask so that students can share their thoughts?

Does each given pyramid have only one grid?

Lesson scenario 2

Lesson title

Volume of a pyramid

Learning outcomes

- Calculates the volume of a pyramid.

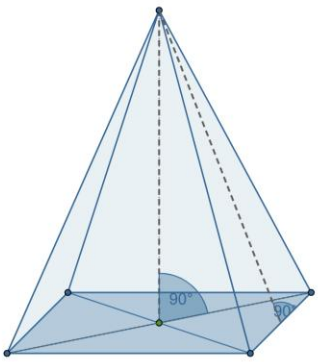
The course of the lesson

Step 1.

Reminder of formulas for the surface areas of flat figures: triangle, parallelogram, trapezoid, regular hexagon.

Step 2.

Derivation of the formula for the volume of a regular pyramid V with edge length a and side wall height length h.

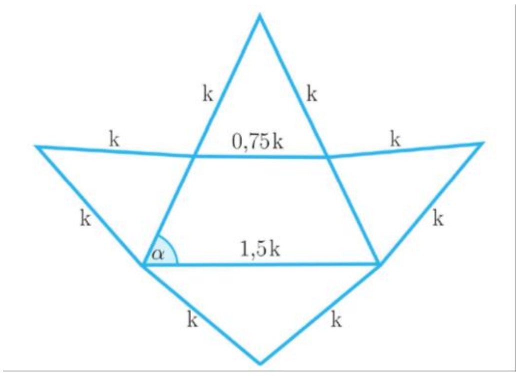


Step 3.

Derivation of the formula for the volume of a regular hexagonal pyramid with edge length a and side wall height length h.

Step 4.

Derivation of the formula for the volume of a pyramid depending on a, α.



What questions should I ask so that students can share their thoughts?

Is it possible to build different regular quadrangular pyramids with the same bases, equal heights and equal volumes?

Suggestions and tips for teachers

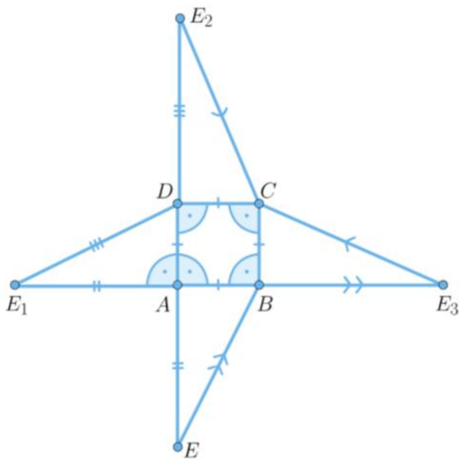
1. Question for students: How will the volume of a pyramid change if the height is tripled?
2. Question for students: How will the volume of a regular quadrangular pyramid change if the height is tripled?
3. Question for students: How will the length of the diagonal of a regular quadrangular pyramid change if the height is tripled?
4. Question for students: How will the total surface area of a regular pyramid change if the height is tripled?
5. You can use the WOLFRAM program to calculate the volume and total surface area of the pyramids.

Worksheets for students

Exercise 1.

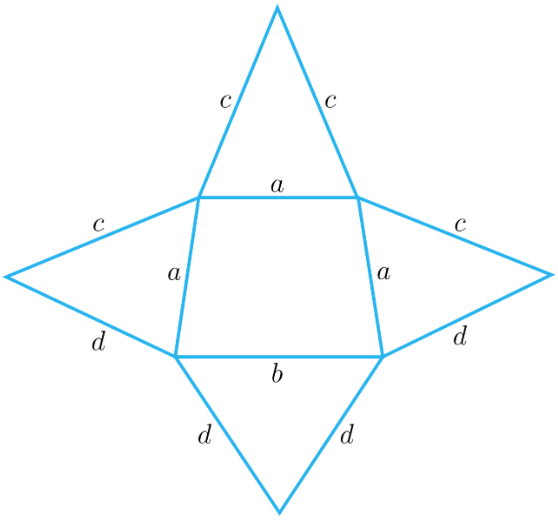
Draw the grid of a pyramid whose base is a square and its faces are right-angled triangles.

Solution



Exercise 2.

Give the name of the pyramid whose grid is shown in the figure.

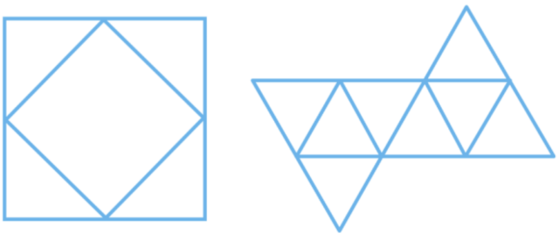


Solution

A quadrangular pyramid with a trapezoid at the base.

Exercise 3.

Which figure shows the grid of a pyramid?

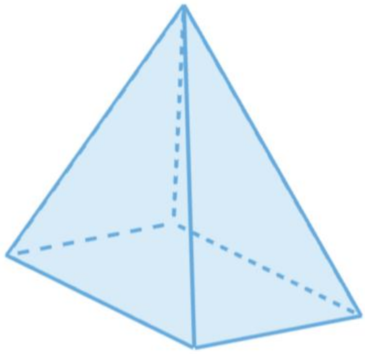


Solution

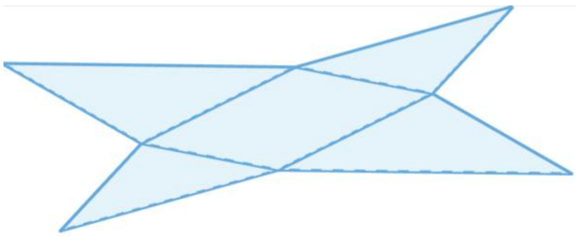
There is no grid of a pyramid in the figures.

Exercise 4.

Draw the grid of the pyramid shown in the figure.



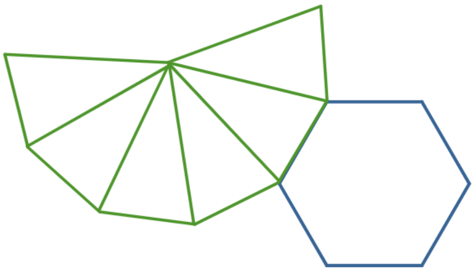
Solution



Exercise 5.

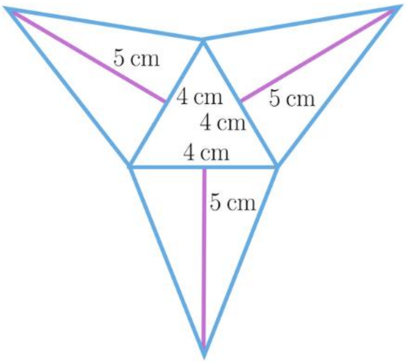
Draw the grid of a regular hexagonal pyramid.

Solution



Exercise 6.

Calculate the length of the height of the pyramid shown in the figure.

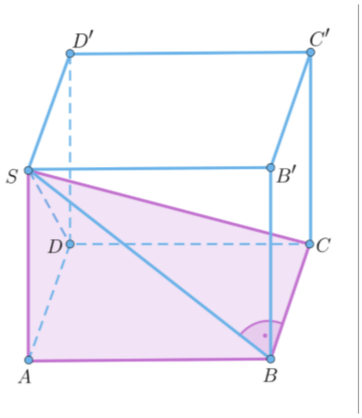


Solution

$h = \sqrt{23\frac{2}{3}}.$

Exercise 7.

Calculate the volume of the pyramid $ABCDS$ if the prism is the cube of the side length of 3.

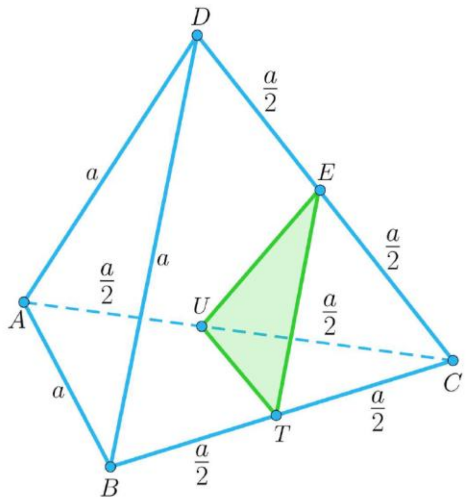


Solution

$V = 9.$

Exercise 8.

A tetrahedron $ABCD$ with edge length 2 is given. Calculate the area of triangle UTE.

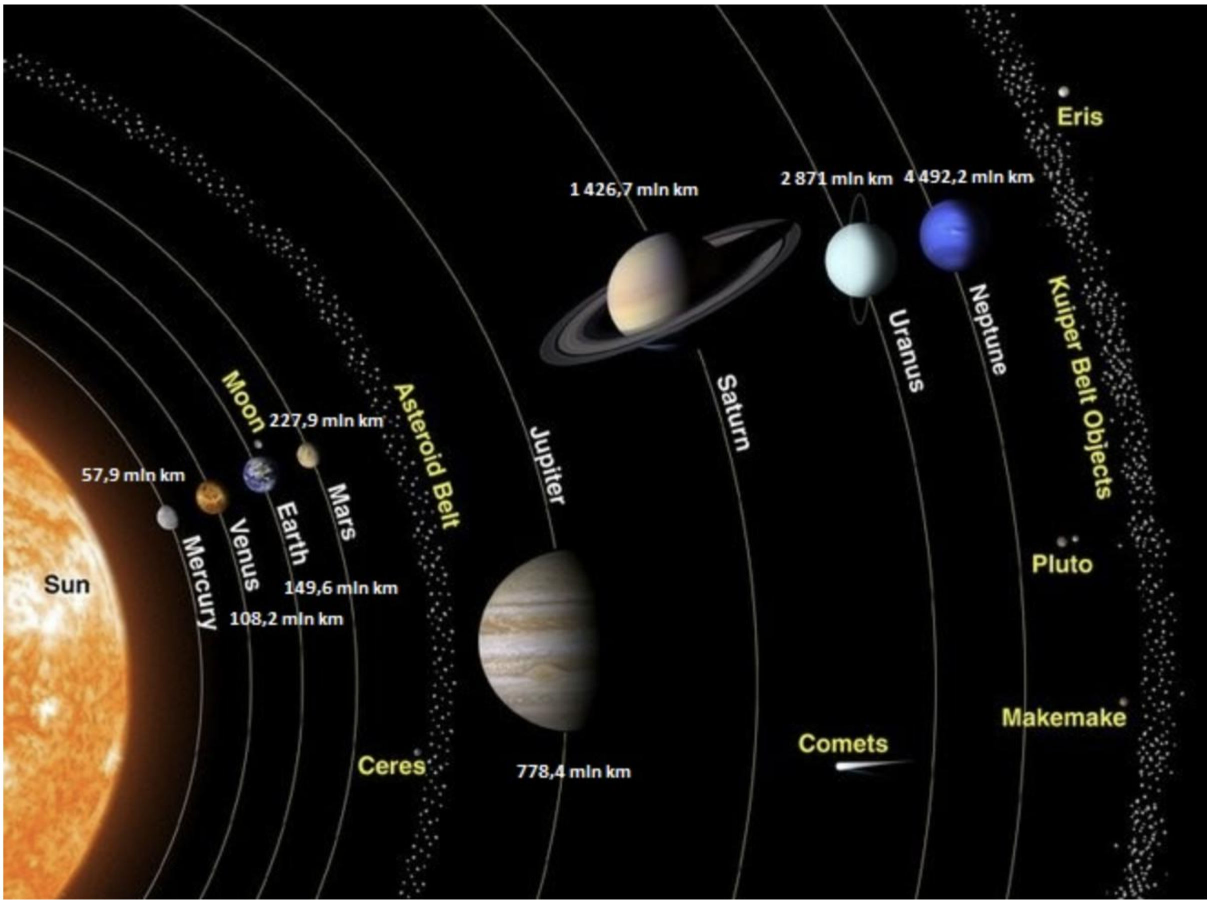


Solution

$P = \sqrt{3}.$

Module 9: Planetary system

Lesson scenarios with VR applications



Lesson scenario 1

Lesson title

Distances in the Solar System

Learning outcomes

- Knows distances in the Solar System.

The course of the lesson

Step 1.

Introduction of the astronomical unit au.

One astronomical unit is the average distance between the Earth and the Sun, which is approximately $149\ 598\ 000\ \mathrm{km}$. For estimated calculations, we can assume $150\ 000\ 000\ \mathrm{km}$.

Step 2.

Determination of the average distance of the Moon from the Earth in the au unit.

Solution. Approximately 0.0026 au.

Step 3.

Defining the astronomical unit light year (ly). A light year is the distance that light travels in a vacuum in a year. $\mathrm{ly} = 63241$ au.

Step 4.

Calculating how many kilometres are in a light year.

Solution. $\mathrm{ly} = 9.5 \cdot 10^{12}$ km.

What questions should I ask so that students can share their thoughts?

The speed of light in a vacuum is $300\ 000\ \frac{\mathrm{km}}{\mathrm{s}}$. How long does it take for light from a vacuum to cross the Earth's equator?

Lesson scenario 2

Lesson title

Quantities comparisons in the Solar System

Learning outcomes

- Compares quantities in the Solar System.

The course of the lesson

Step 1.

The radius of Mars is $3\ 392$ km. The diameter of the Earth is $12\ 756$ km.

Calculation of the surface area of Mars.

Solution. About $1.5 \cdot 10^{8}\ \mathrm{km}^{2}$.

Step 2.

Calculation of the volume of Mars.

Solution. About $1.6 \cdot 10^{11}\ \mathrm{km}^{3}$.

Step 3.

Calculation what part of the Earth's volume is the volume of Mars.

Solution. About 0.15.

Step 4.

Calculation what part of the Earth's surface area is the surface area of Mars.

Solution. About 0.3.

What questions should I ask so that students can share their thoughts?

Compare the densities of Mars and Earth.

Suggestions and tips for teachers

1. Note for students: Become familiar with the astronomical unit – parsec (pc).
2. Question for students: Which planet is closest to the Sun?
3. How long will it take to travel to Mars?
4. You can use the WOLFRAM program for calculations in the Solar System.

Worksheets for students

Exercise 1.

Determine the average distance of Jupiter from the Sun.

Solution

$5.203\ \mathrm{au}$.

Exercise 2.

Determine the average distance of the Earth from the Sun in light years.

Solution

About 8 light minutes.

Exercise 3.

Determine the average distance of the Earth from the Moon in light years.

Solution

About 1.3 light seconds.

Exercise 4.

Determine the approximate value of the Earth's surface area (diameter: $12\ 756\ \mathrm{km}$).

Solution

About $510\ 000\ 000\ \mathrm{km}^2$.

Exercise 5.

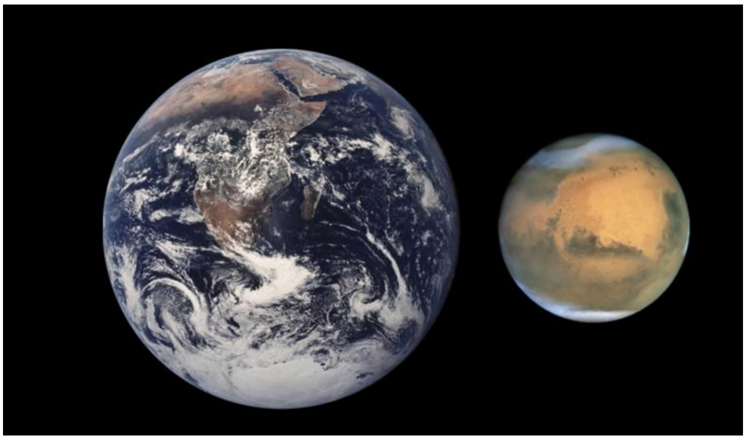
Determine the approximate value of the Earth's volume (diameter: $12\ 756\ \mathrm{km}$).

Solution

About $10^{12}\ \mathrm{km}^3$.

Exercise 6.

The figure shows Earth and Mars to scale. The diameter of the Earth is $12\ 756\ \mathrm{km}$. Estimate the diameter of Mars.



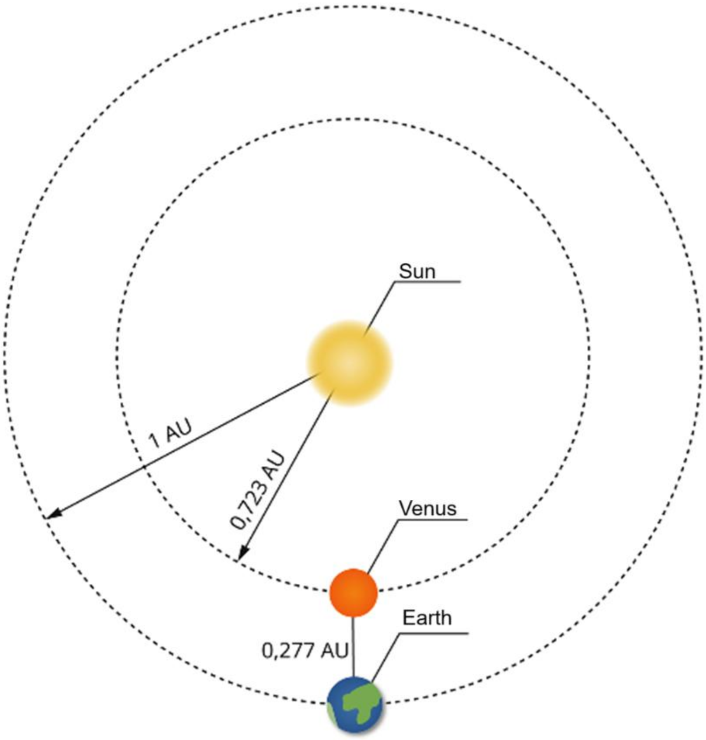
pl.wikipedia.org/wiki/Mars

Solution

About $6\ 800\ \mathrm{km}$.

Exercise 7.

Determine the distance in kilometres of Venus from the Sun.



Module 10: Exploring the Solar System

Lesson scenarios with VR applications

Lesson scenario 1

Lesson title

Space exploration – basic concepts

Learning outcomes

- Uses basic concepts about the space exploration.

The course of the lesson

Step 1.

Introduction of the definition of a spacecraft. Types of spacecraft.

Step 2.

Discussion of the issue of cosmic radiation.

Step 3.

Discussion of the issue of gravity. Gravity on Earth and other planets.

Step 4.

Discussion of the impact of gravity on human health.

What questions should I ask so that students can share their thoughts?

What is the current legal status of space?

Lesson scenario 2

Lesson title

Conquest of space

Learning outcomes

- Discusses the conquest of space.

The course of the lesson

Step 1.

Discussion of the story of the first man in space.

Step 2.

Discussion of the future of humanity in space. Review of popular science articles.

Step 3.

Discussion of the ethical issues of space conquest. Review of popular science articles.

What questions should I ask so that students can share their thoughts?

When was the first human landing on the Moon?

Suggestions and tips for teachers

1. Question for students: Gravity on Earth is $9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}$. Is gravity the same on Mars as on Earth?
2. Question for students: Where is the greatest gravity on Earth?
3. What are NASA's current space research plans?

Worksheets for students

Exercise 1.

Present your opinion on the conquest of the planets of the Solar System.

Exercise 2.

Present your positive arguments about the conquest of the planets of the Solar System.

Exercise 3.

Present your negative arguments about the conquest of the planets of the Solar System.

Exercise 4.

What are the recent discoveries made by space probes in the Solar System?

Exercise 5.

What are the dangers of space debris?

Exercise 6.

On Earth, a person weighs $50\ \mathrm{kg}$. How much weight will this person weigh on Mars?

Solution

About $18\ \mathrm{kg}$.

Module 11: Geometrical interpretation of partial derivatives

Lesson scenarios with VR applications

Lesson scenario 1

Lesson title

Geometric interpretation of partial derivatives

Learning outcomes

- Interprets partial derivatives.

The course of the lesson

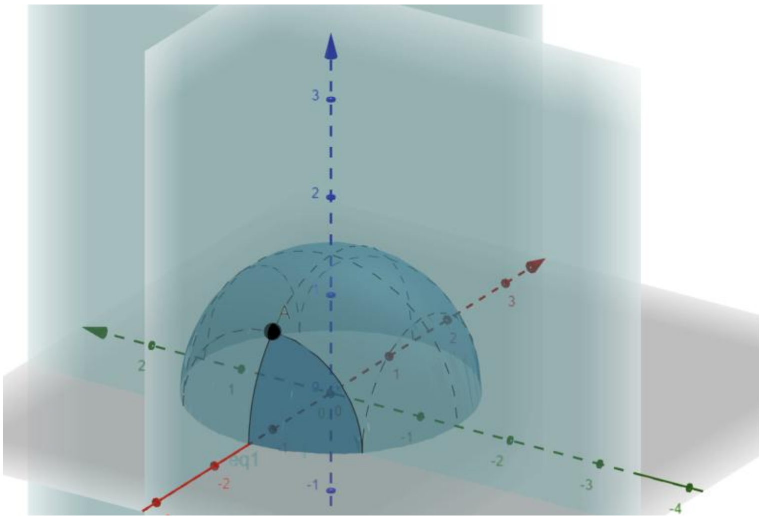
Step 1.

Reminder of the interpretation of the derivative of a function of one variable.

Step 2.

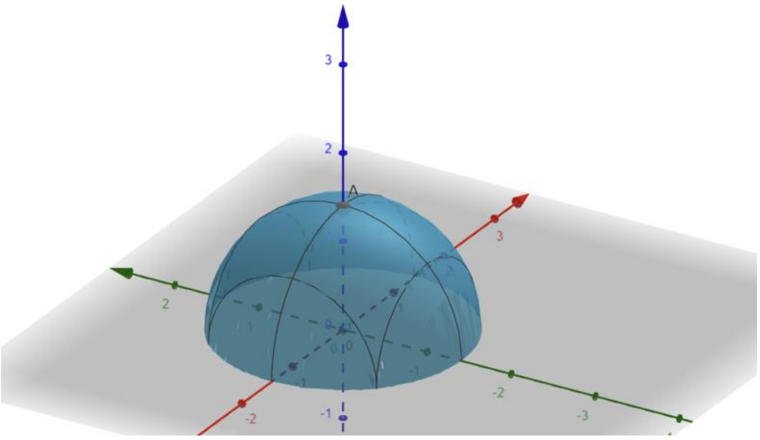
Introduction of the definition of partial derivative and its interpretation for functions of two variables.

Estimation of partial derivatives at a selected point.



Step 3.

Estimation of partial derivatives at a selected point.



Step 4.

Hypothesizing the relationship between the extremum of a function and partial derivatives.

What questions should I ask so that students can share their thoughts?

How to introduce the definitions of partial derivatives for an n-dimensional function?

Lesson scenario 2

Lesson title

Calculating partial derivatives

Learning outcomes

- Calculates partial derivatives.

The course of the lesson

Step 1.

Calculating from the definition the partial derivative for the function $f(x, y) = y + x^2$ with respect to the variable x.

Step 2.

Calculating from the definition the partial derivative for the function $f(x, y) = y + x^2$ with respect to the variable y.

Step 3.

Calculating partial derivatives for the function $f(x, y) = \sin(x^3 + y^2)$.

Step 4.

Calculating partial derivatives for the function $f(x, y) = x \cdot \sin(x^3 + y^2)$.

Step 5.

Calculating partial derivatives for the function $f(x, y) = x \cdot \frac{\sin(x^3+y^2)}{x^2+2}$.

What questions should I ask so that students can share their thoughts?

How to calculate higher-order derivatives?

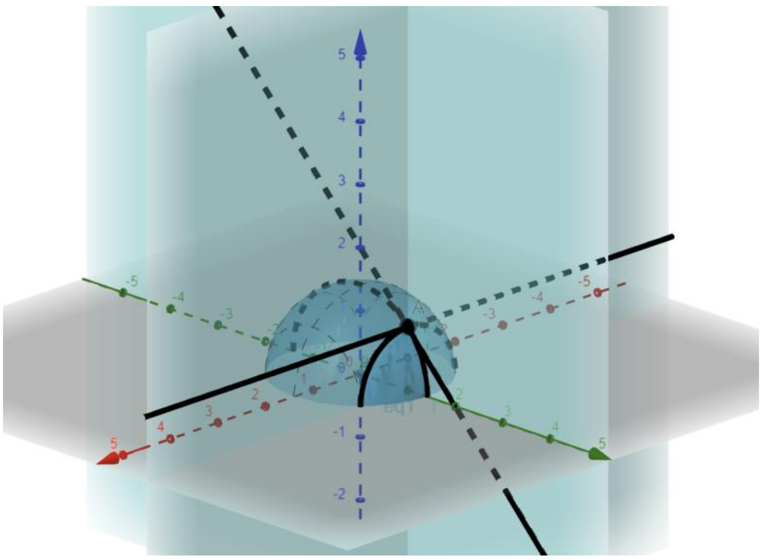
Suggestions and tips for teachers

1. Note for students: When calculating the partial derivative with respect to a selected variable, we calculate it in the same way as for a function of one variable, we treat the remaining variables as constants.
2. Question for students: How to calculate the partial derivative with respect to the variable x a function of three variables? Apply the above rule and calculate $\frac{\partial f(x,y,z)}{\partial x}$ if $f(x, y, z) = x^2zy + x$.
3. The function $f(x, y) = \begin{cases} \frac{xy}{x^2+y^2}, & (x, y) \neq (0,0) \\ 0, & (x, y) = (0,0) \end{cases}$ is not continuous and has partial derivatives at the point $(0, 0)$.
4. Important: the function must not be continuous if partial derivatives exist.
5. You can use the WOLFRAM program to calculate partial derivatives.

Worksheets for students

Exercise 1.

Give the partial derivatives at point A for the function shown in the figure.

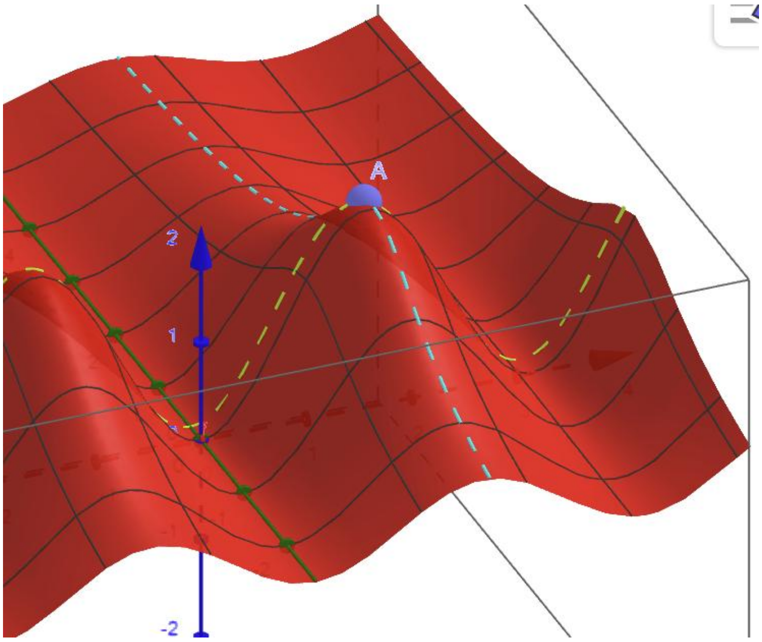


Solution

$\frac{\partial f(x,y)}{\partial x} = 1, \frac{\partial f(x,y)}{\partial y} = 0.$

Exercise 2.

Give the partial derivatives at point A for the function shown in the figure.



Solution

$\frac{\partial f(x,y)}{\partial x} = 0, \frac{\partial f(x,y)}{\partial y} = 0.$

Exercise 3.

Calculate partial derivatives for the function $f(x, y) = x^3y + y^2 + 4$.

Solution

$\frac{\partial f(x,y)}{\partial x} = 3x^2y, \frac{\partial f(x,y)}{\partial y} = x^3 + 2y.$

Exercise 4.

Calculating from the definition the partial derivative for the function $f(x, y) = x^2\, y + x$ with respect to the variable x.

Solution

$\frac{\partial f(x,y)}{\partial x} = \lim_{h \to 0} \frac{f(x+h,y)-f(x,y)}{h} = \lim_{h \to 0} \frac{(x+h)^2y+x+h-(x^2y+x)}{h} = \lim_{h \to 0} \frac{2xhy+h^2}{h} = 2xy.$

Exercise 5.

Calculate partial derivatives for the function $f(x, y) = xe^{x^2+y^2}$.

Solution

$\frac{\partial f(x,y)}{\partial x} = e^{x^2+y^2}(1+2x^2), \frac{\partial f(x,y)}{\partial y} = 2xye^{x^2+y^2}$.

Exercise 6.

Are the domains of the function $f(x,y) = \sqrt{x+y}$ and the partial derivatives the same?

Solution

No.

Exercise 7.

The function is given $f(x,y) = \sin(x^2+y^2)$. Justify that

$$\frac{\partial f(x,y)}{\partial x} + \frac{\partial f(x,y)}{\partial y} = (2x+2y) \cdot f(x,y).$$

Solution

$\frac{\partial f(x,y)}{\partial x} + \frac{\partial f(x,y)}{\partial y} = 2x\sin(x^2+y^2) + 2y\sin(x^2+y^2) = (2x+2y) \cdot f(x,y)$.

Exercise 8.

Give an example of the function $f(x,y) = \sin(x^2+y^2)$ which meets the condition

$$\frac{\partial f(x,y)}{\partial x} + \frac{\partial f(x,y)}{\partial y} = 0.$$

Solution

$f(x,y) = x - y$.

Module 12: Spherical coordinates

Lesson scenarios with VR applications

Lesson scenario 1

Lesson title

Polar coordinates

Learning outcomes

- Uses polar coordinates.

The course of the lesson

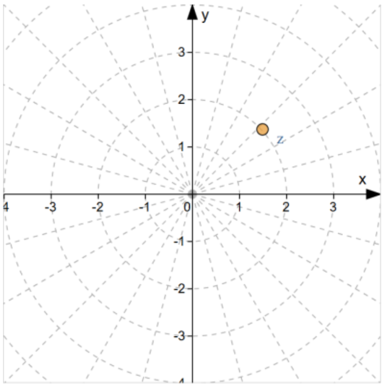
Step 1.

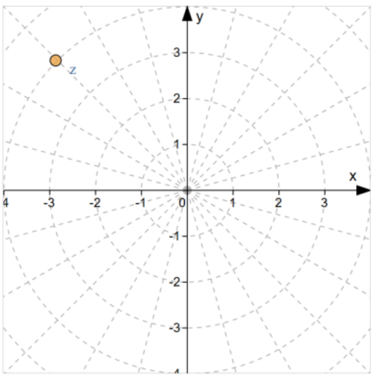
Reminder of the properties and graphs of the functions $\sin(x)$ and $\cos(x)$.

Step 2.

Discussion of the issue of polar coordinates $\begin{cases} x = r\cos(\alpha) \\ y = r\sin(\alpha) \end{cases}$.

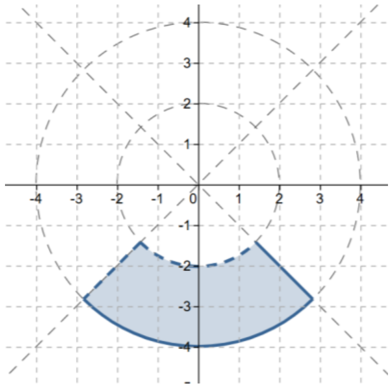
Determining the polar coordinates of the points shown in the figure.





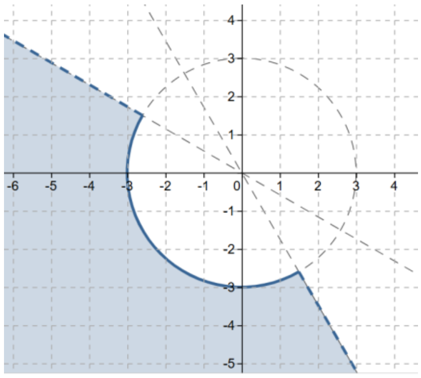
Step 3.

Describing an area using polar coordinates.



Step 4.

Describing an area using polar coordinates.



What questions should I ask so that students can share their thoughts?

How do polar coordinates differ from Cartesian coordinates?

Lesson scenario 2

Lesson title

Spherical coordinates

Learning outcomes

- Uses spherical coordinates.

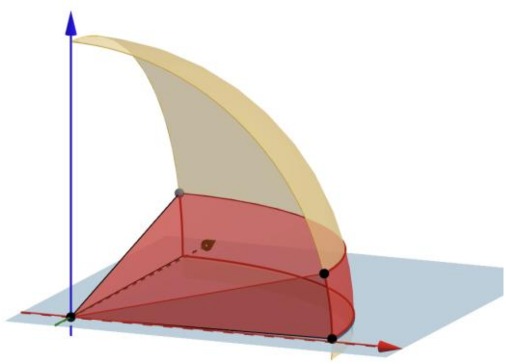
The course of the lesson

Step 1.

Introduction of spherical coordinates $\begin{cases} x = r\cos(\theta)\cos(\alpha) \\ y = r\cos(\theta)\sin(\alpha). \\ z = r\sin(\theta) \end{cases}$

Step 2.

Discussion of sets in spherical coordinates.



$$\begin{cases} 0 \le r \le 4 \\ 0^\circ \le \alpha \le 90^\circ \\ 0^\circ \le \theta \le 20^\circ \end{cases}$$

Step 3.

Calculate the distance between points if given in spherical coordinates.

What questions should I ask so that students can share their thoughts?

What are the limits on spherical coordinate values?

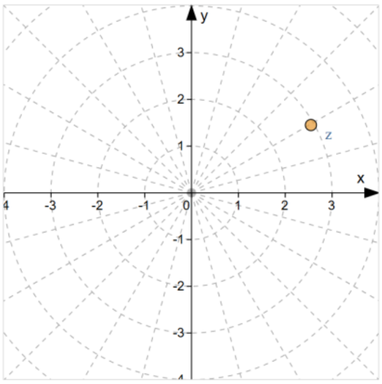
Suggestions and tips for teachers

1. Question for students: How to write the equation of a circle in polar coordinates?
2. Question for students: How to write the equation of a sphere in spherical coordinates?
3. How to convert polar coordinates to Cartesian coordinates?
4. How to convert spherical coordinates to Cartesian coordinates?
5. You can use the WOLFRAM program to calculate spherical coordinates.

Worksheets for students

Exercise 1.

Determine the polar coordinates of the point shown in the figure.

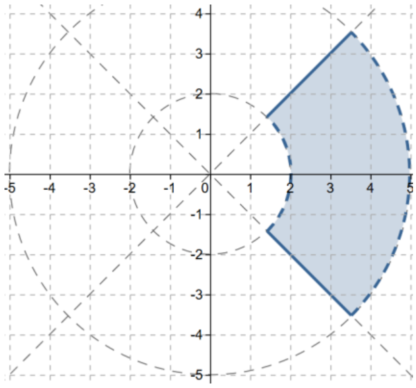


Solution

Polar coordinates: $r = 3, \alpha = 30^\circ$.

Exercise 2.

Describe the set shown in the figure using polar coordinates.



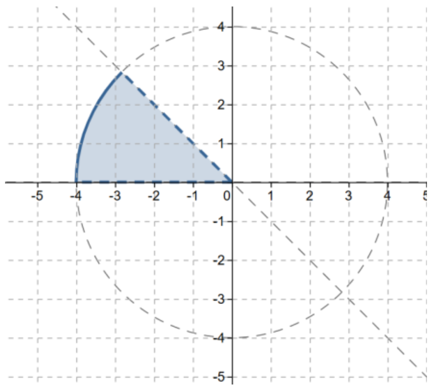
Solution

$$\begin{cases} 2 < r < 5 \\ -45° \leq \alpha \leq 45° \end{cases}.$$

Exercise 3.

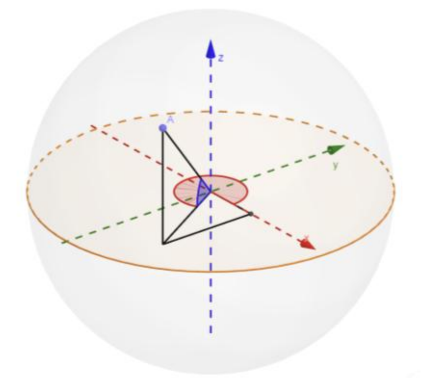
Draw the set $\begin{cases} 0 \leq r \leq 4 \\ 135° < \alpha < 180° \end{cases}$ given in polar coordinates.

Solution



Exercise 4.

Estimate the spherical coordinates based on the figure.



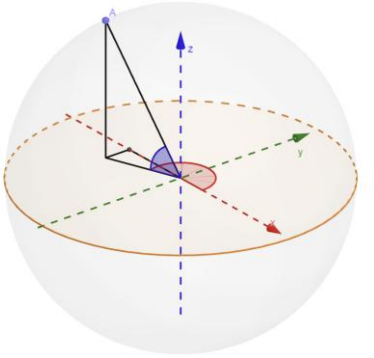
Solution

$\alpha = 300°, \theta = 45°, r = 3.$

Exercise 5.

Draw a point with polar coordinates: $\alpha = 200°, \theta = 60°, r = 3.$

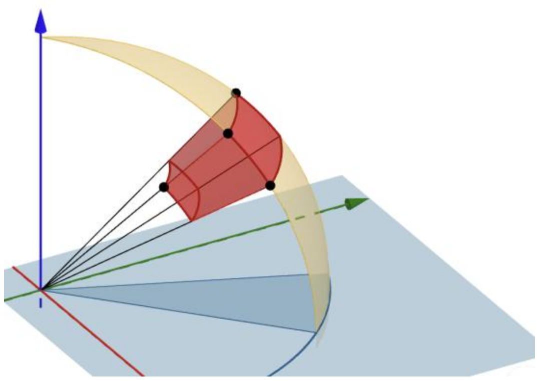
Solution



Exercise 6.

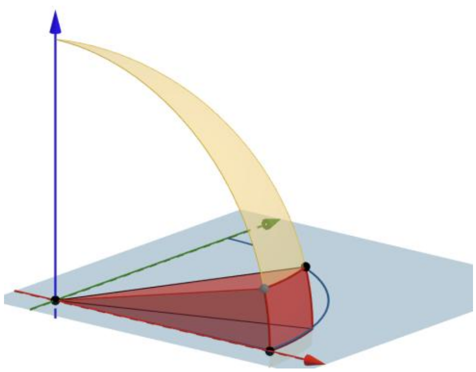
Draw the set $\begin{cases} 2 \leq r \leq 4 \\ 30° \leq \alpha \leq 60°. \\ 40° \leq \theta \leq 50° \end{cases}$

Solution



Exercise 7.

Describe the set using spherical coordinates.



Solution

$$\begin{cases} 0 \le r \le 4 \\ 0^\circ \le \alpha \le 20^\circ. \\ 0^\circ \le \theta \le 20^\circ \end{cases}$$

Module 13: Vectors, operations on vectors, scalar

Lesson scenarios with VR applications

Lesson scenario 1

Lesson title

Geometric interpretation of vectors in three-dimensional space, operations on vectors

Learning outcomes

- Interprets vectors in three-dimensional space.

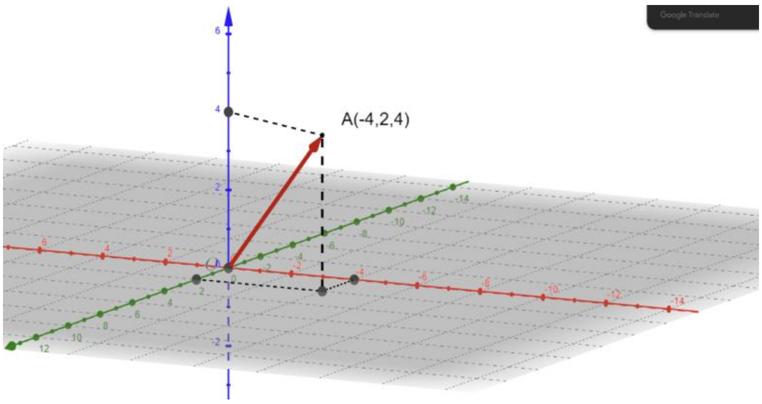
The course of the lesson

Step 1.

Reminder of the interpretation of a vector on a plane and operations on vectors.

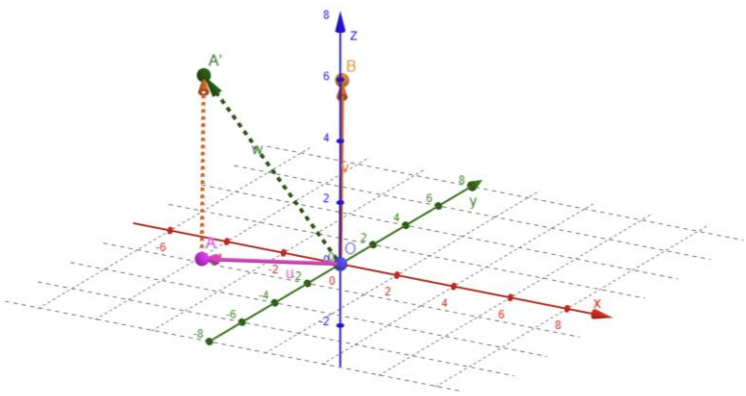
Step 2.

Drawing a vector with coordinates $\vec{v} = [-4,2,4]$.



Step 3.

Presentation of the sum of vectors $\vec{u}, \vec{v}$ if $\vec{u} = [-4,-2,0]$, $\vec{v} = [0,0,6]$.



Step 4.

Performing the operation $\vec{u} + 3\vec{v} - 2\vec{s}$ on the vectors $\vec{u} = [1,3,0]$, $\vec{v} = [-1,1,2]$, $\vec{s} = [1,0,0]$.

Solution: $\vec{u} + 3\vec{v} - 2\vec{s} = [-4,6,6]$.

What questions should I ask so that students can share their thoughts?

How to define vectors, operations on vectors in n-dimensional space?

Lesson scenario 2

Lesson title

Scalar product, vector product in three-dimensional space

Learning outcomes

- Calculates scalar and vector products in three-dimensional space.

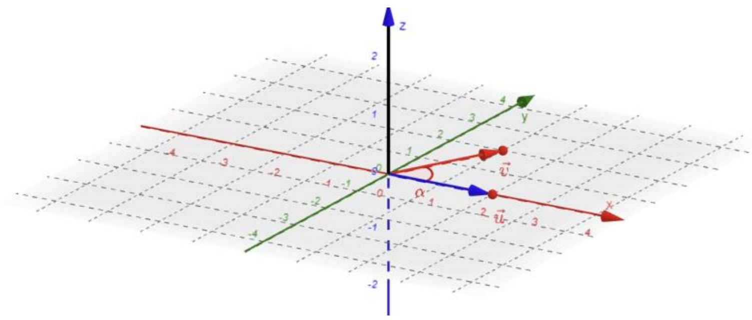
The course of the lesson

Step 1.

Reminder of the interpretation of a scalar product on a plane.

Step 2.

Graphical representation of the interpretation of the vector product for vectors $\vec{u}, \vec{v}$ in a space.



Step 3.

Calculating the scalar product in a space for vectors $\vec{u} = [1,4,0]$, $\vec{v} = [3,1,1]$.

Step 4.

Introduction of the definition and interpretation of a vector product.

What questions should I ask so that students can share their thoughts?

How to define scalar product and vector product in n-dimensional space.

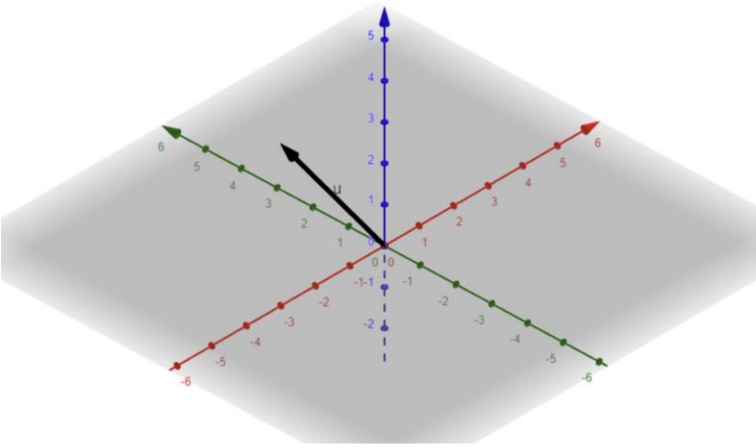
Suggestions and tips for teachers

1. What are the practical applications of the scalar product?
2. What are the practical applications of the vector product, e.g. in computer graphics?
3. Discuss the basic properties of the scalar product.
4. Discuss the basic properties of a vector product.
5. How to check that vectors are parallel or perpendicular?
6. You can use the WOLFRAM program to calculate the scalar or vector product.

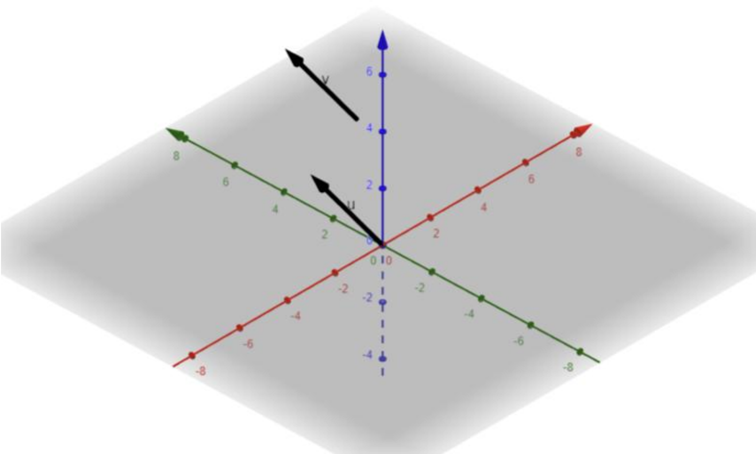
Worksheets for students

Exercise 1.

Vectors $\vec{u}, \vec{v}$ are equal. Draw the vector $\vec{v}$ knowing that $\vec{u} = [-2,1,3]$.

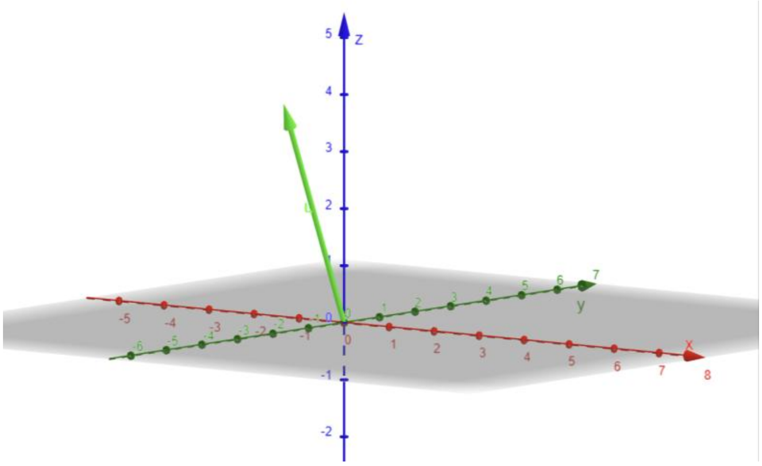


Solution

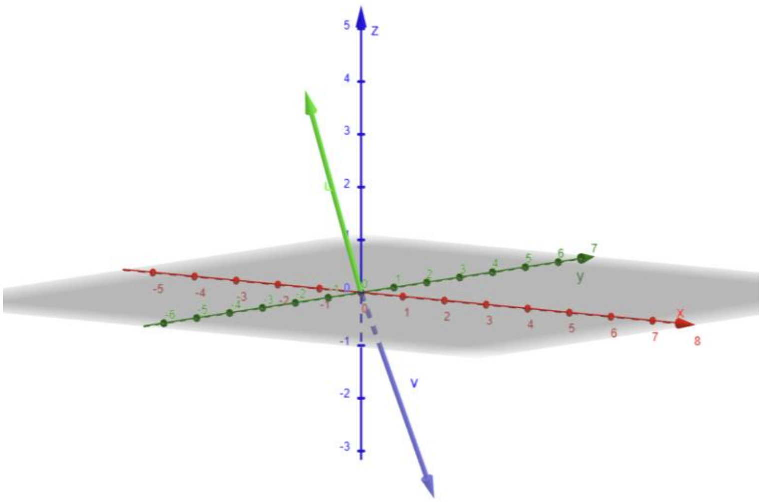


Exercise 2.

Vectors $\vec{u}, \vec{v}$ are opposite. Draw the vector $\vec{v}$ knowing that $\vec{u} = [0,2,4]$.



Solution



Exercise 3.

Calculate the coordinates of the vector $\overrightarrow{AB}$ if $A = (2,5,-1)$, $B = (0,2,4)$.

Solution

$\overrightarrow{AB} = [-2,-3,5]$.

Exercise 4.

Calculate the length of the vector $\vec{u} = [3,4,0]$.

Solution

$|\vec{u}| = 5$.

Exercise 5.

Determine the parameters a, b, c so that the vectors $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are equal if $A = (2,5,-1)$, $B = (0,2,4)$, $C = (a,0,c)$, $D = (0,b,4)$.

Solution

$a = 2, b = -3, c = -1.$

Exercise 6.

Calculate the vector product of vectors $\vec{u} = [1,4,0]$, $\vec{v} = [3,1,1]$.

Solution

$\vec{u} \times \vec{v} = [4, -1, -11].$

Exercise 7.

Calculate the scalar product of vectors $\vec{u} = [3,4,0]$, $\vec{v} = [2,4,1]$.

Solution

22.

Exercise 8.

Check if the triangle is right-angled $A = (2,5,-1)$, $B = (0,2,10)$, $C = (0,0,0)$.

Solution

Yes.

Exercise 9.

Prove that $\vec{u} \cdot \vec{u} = |\vec{u}|^2$.

Solution

Let's assume that $\vec{u} = [a, b, c]$. Then $\vec{u} \cdot \vec{u} = [a, b, c] \cdot [a, b, c] = a^2 + b^2 + c^2 = |\vec{u}|^2$.

Exercise 10.

Prove that $\vec{u} \times \vec{u} = \vec{0}$.

Solution

Let's assume that $\vec{u} = [a, b, c]$. Then $\vec{u} \times \vec{u} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a & b & c \\ a & b & c \end{vmatrix} = \vec{0}$.

Exercise 11.

Calculate the volume of the solid defined by the vectors $\vec{u} = [1,4,0]$, $\vec{v} = [3,1,1]$, $\vec{s} = [1,1,0]$.

Solution

$\vec{u} \times \vec{v} = [4, -1, -11]$. $V = \vec{s} \cdot (\vec{u} \times \vec{v}) = 3$.